MADURAI KAMARAJ UNIVERSITY

(University with Potential for Excellence)

DIRECTORATE OF DISTANCE EDUCATION

B.Sc., Physics

PAPER - I

MEHANICS AND PROPERTIES OF MATTER

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B. Sc. PHYSICS - I Year

Paper – I – Mechanics and Properties of Matter

Unit – I

Newton's law of motion – Force – Mass - Momentum and Impulse, Law of Conservation of Linear Momentum – Collision - Elastic and Inelastic collision-Newton's law of impact. Coefficient of restitution-Impact of moving sphere on a fixed plane-Direct and Oblique impact of moving two smooth spheres - Calculation of final velocities - Laws of Kinetic energy - Projectile motion - Frictional forces-Center of mass of solid objects-Conservation of Momentum in a system of particles.

Unit –II

Uniform circular motion - The dynamics of uniform circular motion - Moment of inertia of circular disc about an axis passing through its center and perpendicular to its plane, through its diameter, through its tangent - Moment of inertia of a solid sphere about all axes - Angular momentum and angular velocity and torque - Relation between angular momentum and torque-Kinetic energy of rotation and the work energy theorem-Conservation of angular momentum - Work done by constant force-work done by variable force - work and kinetic energy in rotational motion – Expression for the acceleration of a body rolling down an inclined plane

Unit – III

Gravitation – Newton's law of gravitation – Kepler's law of planetary motion – mass of earth Gravitational field and potential at a point inside and outside a spherical of earth– Mass and density of earth – Determination of G (Boy's method) – Variation of 'g' with altitude, depth and latitude – Earthquake – seismograph – modern application of seismology - Satellites – Orbital velocity – Escape velocity – Stationary satellite – Jet plane – Rocket – Principle, theory _ Velocity of rocket at any instant – Rocket Propulsion – specific impulse – multistage rocket – Shape of the rocket.

Unit – IV

Elasticity – Stress, strain – Poisson's ratio – Hooke's Law – Moduli of Elasticity – Young's modulus, Bulk modulus, rigidity modulus – Bending of a beam – Bending moment – Uniform and Non-uniform bending – Theory and experiment – Determination of Poisson's ratio – Torsional Pendulum – Determination of co-efficient of rigidity (□) for a wire I – section griders.

Unit V

Fluids – Flow of a fluid – Rate of flow – Viscosity – Coefficient of Viscosity – Critical velocity – Laminar and Vortex flow – Poiseuille equation for flow of liquid through a tube – Experimental determination of 'n' – Poiseuille's method and Stoke's method – Ostwald Viscometer – Determination of gases – Rankine's method for the determination for the viscosity of a gas – Surface tension – Free energy of a surface and surface tension – Excess pressure inside a liquid drop and inside a soap bubble – Work done in blowing a bubble – Angle of contact – Capillary rise – Pilot tube and Venturi meter – Bernoulli's theorem.

Text Books:

1. Mechanics and Properties of Matter by R. Murugesan – Retd. Prof., Vivekananda College, Thiruvedagam West.
2. David Halliday, Robert Resnick, Kenneth S. Kran 2002 , fifth edition, volume 1, physics, John Wiley and Sons, INC.

Paper – I – Mechanics and Properties of Matter

UNIT–I LAWS OF MOTION, COLLISION AND PROJECTILES

Structure

1.1 Introduction

In our everyday life, we observe that some effort is required to put a stationary object into motion or to stop a moving object. Normally we have to apply a force to change the state of rest or of motion of an object.

According to Aristotle, a constant force has to be applied on a body so as to keep it in motion with a constant velocity. Later on, Galileo stated that no force is required for a body to move with a uniform velocity. Newton was the first man to formulate the laws concerning the state of rest and motion of objects. There are three laws of motion.

We learn much about atomic, nuclear and elementary particles experimentally by observing collision between them. The study of collision is based on the principle of conservation of momentum and conservation of energy.

1.2 Objective

By going through this chapter, students are able to

- analyze situations in which a particle remains at rest, or moves with constant velocity, under the influence of several forces
- understand how Newton's Second Law applies to an object subject to forces such as gravity, the pull of strings, or contact forces
- understand conservation of linear momentum
- understand the significance of the coefficient of friction
- recognize and solve problems that call for application both of conservation of energy and Newton's Laws
- understand different types of collisions, calculate final velocities after collision and loss of energy due to collision
- understand the technique for finding center of mass

1.3 Newtons laws of motion

FIRST LAW: Everybody remains in the state of rest or of uniform motion in a straight line unless compelled by impressed forces to change that state.

SECOND LAW: The rate of change of momentum of an object is directly proportional to the impressed force and takes place in the direction of the force.

THIRD LAW: For every action there is always an equal and opposite reaction.

These three laws form the fundamental basis of dynamics. The first law gives the definition of force. It is also called as '**law of inertia**'. Inertia is the inability of a material body to change by itself its state of rest or of uniform motion in a straight line. The second law gives a measure of a force. It states that Force = mass x acceleration. The third law defines the nature of force and specifies the property of force. According to this law action and reaction are equal in magnitude and opposite in direction and act on two different bodies of an isolated system. For example, when a gun is fired it exerts forward force on the bullet (action). The bullet exerts equal and opposite force (reaction) on the gun. This results in the recoil of the gun.

1.4 Force and mass

Force is defined as that external agency that changes or tends to change the state of rest or of uniform motion of a body in a straight line. No one has seen, felt or tasted a force. However, we can always see or feel the effect of a force.

Force is a mathematical concept. By definition, force is equal to the time rate of change of momentum of a particle. If p is the momentum of a given particle then,

$$Force = \frac{dp}{dt} \qquad \text{..... (1)}$$

If a body of mass 'm' is moving with a velocity 'v' then the momentum

$$p = mv \qquad \text{..... (2)}$$

Differentiating (2) with respect to time

$$\frac{dp}{dt} = \frac{d(mv)}{dt}$$

$$= m\frac{dv}{dt} + v\frac{dm}{dt}$$

But $\frac{dm}{dt} = 0$ as m is constant

$\frac{dp}{dt} = m\frac{dv}{dt} = m\,a$ where 'a' is the acceleration of the particle.

According to Newton's second law of motion, the rate of change of momentum of a body is directly proportional to the impressed force.

i.e. $F \propto \frac{dp}{dt}$

$F \propto m\,a$

Or $F = k\,m\,a$

The units of m, a and F are chosen that the constant k = 1.Therefore,

$$F = m\,a$$

i.e. Force = mass x acceleration.

This shows that Newton's second law gives a measure of force.

Unit of force: In SI system, unit of force is **newton** and is denoted by 'N.

$F = m\,a$, if $m = 1 kg$ and $a = 1\ m/s^2$ then $F = 1$ newton.

One **newton** is defined as the force which produces acceleration of $1\ m/s^2$ on a mass of 1 kg.

The **dimensions** of force are $[MLT^{-2}]$.

MASS: Mass is the amount of matter present in a body (or) is a measure of how much matter an object has.

Mass of an object is also defined as the measure of its inertia. Inertia is the inability of the object to change its state of rest or of uniform motion by itself. For example, if we kick a football, it flies away. But if we kick a stone of the same size with equal force, it hardly moves. We may, in fact get an injury in our foot. We say that stone has more inertia than football. The inertia of an object is measured by its mass. In SI system, the unit of mass is kilogram (kg).

1.5 Momentum and impulse

Momentum

Let us consider some observations from our everyday life. During the game of table tennis, if a ball hits a player it does not hurt him. On the other hand when fast moving cricket ball hits a player it may hurt him. A moving truck with very low speed may hurt a person standing in its path. A small mass such as a bullet may kill a person when fired from a gun with a higher velocity. These observations suggest that the impact produced by an object depends on its mass and velocity. Newton introduced a property, which combines both mass and velocity of an object called momentum.

The momentum **p** of an object is defined as the product of its mass **m** and velocity **v**. That is, $\mathbf{p} = \mathbf{m\,v}$. It is also called as linear momentum. It is a vector quantity and it has the same direction as the velocity. In SI system momentum is expressed in $kg\ ms^{-1}$.

Impulse

Let a constant force **F** act on a body for a time **t.** then the product **F x t** is called as the impulse of the force. It is denoted by **I.**

i.e. $$Impulse = Force \times time = F \times t$$

The impulse is equal to the change in momentum of a body. According to Newton's second law of motion, F = m a, where 'm' is the mass of the body and 'a' is the acceleration produced. We know that acceleration is the rate of change of velocity. That is, $a = \frac{(v-u)}{t}$ where u and v are the initial ad final velocities of the body. Hence, $F = m\frac{(v-u)}{t}$

Therefore, impulse $I = F \times t = m(v-u) = mv - mu$

Impulse = final momentum – initial momentum

i.e. Impulse = change in momentum.

Thus the impulse of a force is change of momentum produced in the body. Impulse is a vector quantity whose direction is that of the force. The unit of impulse is Ns (Newton second).

1.6 Law of conservation of linear momentum

The law of conservation of linear momentum states that, in the absence of external impressed force, the total momentum of a system of objects unchanged or conserved.

Consider two particles in an isolated system. These two particles may interact with each other. . The force of interaction may be gravitational, magnetic or electric in origin. Suppose that no external force is acting on the system. In such a case, the momentum of each particle may change due to interaction but the momentum of the system will remain constant. The change in momentum of one particle in any interval of time is equal in magnitude and opposite in direction to the change in momentum of the other particle. Hence the net change in momentum of the system is zero in the absence of external force. That is, the total momentum of the system remains constant. This is known as the principle of conservation of linear momentum.

According to Newton's second law of motion, $F = \frac{d(mv)}{dt}$ in the direction in which F =0, we have, $\frac{d(mv)}{dt} = 0$ (or) mv = a constant. Thus in the direction in which there is no component of impressed force, the total momentum in that direction is conserved and is constant throughout the motion.

1.7 Collisions

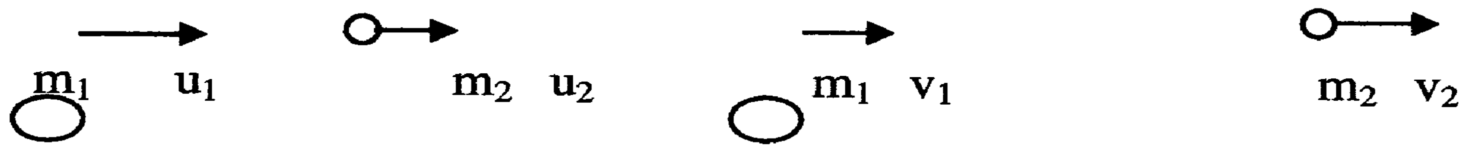
A **collision** is an isolated event in which two or more moving bodies exert forces on each other for a relatively short time. It is a short duration interaction between two bodies or more than two bodies, causing change in motion of bodies involved. The change in motion is due to internal forces acted between them during collision. In collision, a relatively large force acts on colliding particles for a relatively short time. The force is called an impulsive force. All collisions conserve momentum.

Principle of conservation of momentum: 'the momentum of a system of particles before collision is equal to the momentum of the system after the particles collide'.

1.7.1 Elastic and inelastic collisions

In all collisions momentum is conserved. Hence, collisions are distinguished according to whether the kinetic energy of the system is conserved or not. Obviously, there are two types of collision (i) elastic and (ii) inelastic. If the kinetic energy of the system is conserved, the collision is called elastic collision. If the kinetic energy of the system is not conserved, the collision is said to be inelastic collision.

Elastic collisions are those in which the total kinetic energy before and after the collision remains unchanged. Collisions between atomic, nuclear and fundamental particles are the true elastic collisions. Collisions between ivory or glass balls can be treated as approximately elastic collisions. If the kinetic energy before collision is not equal to the kinetic energy after collision, then the collision is called **inelastic collision**. If the two particles stick together after collision, the collision is said to be completely inelastic. For example, the collision between the bullet and the target is completely inelastic when the bullet remains embedded to the target.



Consider two particles of masses 'm_1' and 'm_2' moving with velocities 'u_1'and 'u_2'. They collide with each other and there velocities are 'v_1'and 'v_2'after collision. Momentum is conserved in all collision.

Momentum before collision = momentum after collision

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2 \qquad ...... (1)$$

If the kinetic energy is conserved,

i.e., kinetic energy before collision = kinetic energy after collision

$$\tfrac{1}{2} m_1u_1^2 + \tfrac{1}{2} m_2u_2^2 = \tfrac{1}{2} m_1v_1^2 + \tfrac{1}{2} m_2v_2^2 \quad ...... (2)$$

Then the collision is called elastic collision. On the other hand, if the kinetic energy is conserved,

i.e., kinetic energy before collision ≠ kinetic energy after collision

$$\tfrac{1}{2} m_1u_1^2 + \tfrac{1}{2} m_2u_2^2 \neq \tfrac{1}{2} m_1v_1^2 + \tfrac{1}{2} m_2v_2^2 ...... (3)$$

Then the collision is called inelastic collision.

1.7.2 One dimensional elastic collision of two spheres

Consider two particles of masses 'm_1' and 'm_2' moving initially with velocities 'u_1'and 'u_2'in the same direction. They collide with each other and after collision let their velocities are 'v_1'and 'v_2'. Then from the law of conservation of momentum,

Initial momentum before collision = Final momentum after collision

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2 \qquad \text{...... (1)}$$

Since the collision is elastic, from the law of conservation of kinetic energy, kinetic energy before collision = kinetic energy after collision

$$\tfrac{1}{2} m_1u_1^2 + \tfrac{1}{2} m_2u_2^2 = \tfrac{1}{2} m_1v_1^2 + \tfrac{1}{2} m_2v_2^2 \qquad \text{...... (2)}$$

Equation (1) can be written as

$$m_1 (u_1 - v_1) = m_2 (v_2 - u_2) \qquad \text{...... (3)}$$

Equation (2) can be written as

$$m_1 (u_1^2 - v_1^2) = m_2 (v_2^2 - u_2^2) \qquad \text{...... (4)}$$

Dividing equation (4) by equation (3) and assuming that $u_1 \neq v_1$ and $u_2 \neq v_2$

$$\frac{m_1 (u_1^2 - v_1^2)}{m_1 (u_1 - v_1)} = \frac{m_2 (v_2^2 - u_2^2)}{m_2 (v_2 - u_2)}$$

Or

$$u_1 + v_1 = u_2 + v_2 \qquad \text{...... (5)}$$

$$v_2 - v_1 = -(u_2 - u_1)$$

i.e. Relative velocity after collision = - (Relative velocity before collision)

To find v_1 and v_2

From equation (5) $v_2 = u_1 + v_1 - u_2$

Substituting this in equation (3) we get,

$$m_1 (u_1 - v_1) = m_2 (u_1 + v_1 - 2u_2)$$

$$v_1 (m_1 + m_2) = (m_1 - m_2) u_1 + 2m_2u_2$$

Or

$$v_1 = \frac{(m_1 - m_2)}{(m_1 + m_2)} u_1 + \frac{2 m_2 u_2}{(m_1 + m_2)} \qquad \text{.... (6)}$$

Similarly from equation (5), $v_1 = v_2 + u_2 - u_1$

Substituting this in equation (3) we get,

$$m_1 [u_1 - (v_2 + u_2 - u_1)] = m_2 (v_2 - u_2)$$

Simplifying $v_2 = \frac{2m_1}{(m_1 + m_2)} u_1 + \frac{2(m_2 - m_1)}{(m_1 + m_2)} u_2$ ...... (7)

1.8 Fundamental principles of impact

1.8.1 Newton's law of impact and Coefficient of restitution:

Newton studied experimentally the impact of two smooth elastic bodies and proposed the following law.

When two bodies impinge directly, their relative velocity after impact is in a constant ratio to their relative velocity before impact and is in the opposite direction. This constant ratio depends only on the material of the bodies and not on their masses or velocities. If u_1, u_2 be the velocities of two bodies before the impact and v_1, v_2 the velocities after impact, then

Relative velocity after impact $= v_1 - v_2$
Relative velocity before impact $= u_1 - u_2$

$$\frac{v_1 - v_2}{u_1 - u_2} = - \text{(constant)}$$

This constant is called the **coefficient of restitution** and is denoted by the letter *e*.

$$\frac{v_1 - v_2}{u_1 - u_2} = - e$$

or $\quad v_1 - v_2 = - e\,(u_1 - u_2)$

Definition of coefficient of restitution:

The ratio (with a negative sign) of the relative velocity of two bodies after impact to their relative velocity before impact is called the coefficient of restitution.

$$- e = \frac{v_1 - v_2}{u_1 - u_2}$$

The value of e lies between 0 and 1. If e=0, the bodies are called perfectly plastic bodies. If e=1, the bodies are called perfectly elastic bodies. For two glass balls, e=0.94; for two lead balls, e=0.2.

1.8.2 Motion of two smooth bodies' perpendicular to the line of impact:

When two smooth bodies impinge, there is no tangential action between them. Hence there is no change of momentum along the common tangent. Hence, there is no change of velocity for either body along the tangent. In other words, there is no change in the velocity of a body in a direction perpendicular to the common normal due to impact.

1.8.3. Principle of conservation of momentum:

The total momentum of two bodies after impact along the common normal should be equal to the total momentum before the impact along the same direction.

The above three principles are sufficient to determine the change in motion of two impinging smooth bodies.

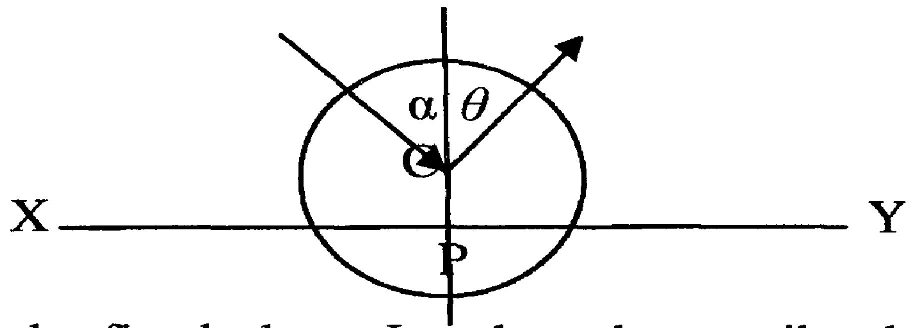
Definitions:

i) Two bodies are said to impinge **directly** when the direction of motion of each is along the common normal at the point where they touch.

ii) Two bodies are said to impinge **obliquely** if the direction of motion of either or both is not along the common normal at the point of contact.

iii) The common normal at the point of contact is called the line of I impact. Thus, in the case of two spheres the line of impact is the line joining their centers.

1.9 Oblique impact of a smooth sphere on a fixed smooth plane



Let XY be the fixed plane. Let the sphere strike the fixed plane at point P. Then if C is the centre of the sphere, CP is the common normal at the point of contact of the plane and the sphere. Let u and v be the velocities of the sphere before and after impact making angles α and θ respectively with the common normal CP (Fig). By Newton's experimental law, the relative velocity of the sphere along the common normal after impact is $-e$ times its relative velocity along the common normal before impact.

$$v \cos\theta - 0 = -e(-u \cos\alpha - 0)$$

or $$v \cos\theta = e\, u \cos\alpha \quad \text{.... (1)}$$

Since both the sphere and the plane are smooth, there is no force in a direction parallel to the plane. Hence the velocity of the sphere resolved parallel to the plane is unaltered by the impact.

$$\therefore v \sin\theta = u \sin\alpha \quad \text{....(2)}$$

Dividing (1) by (2), $\cot\theta = e \cot\alpha$ .... (3)

Squaring and adding (1) and (2),

$$v^2 = u^2(\sin^2\alpha + e^2\cos^2\alpha) \quad \text{.....(4)}$$

Equation (4) and (3) give the velocity and direction of the sphere after impact.

Cor.1. The impulse of the plane on the sphere is measured by the change of momentum of the sphere measured along the normal.

$$I = m[v\cos\theta - (-u\cos\alpha)] = m[v\cos\theta + u\cos\alpha]$$

$$= m[e\,u\cos\alpha + u\cos\alpha]$$

$$\therefore \quad I = mu(1+e)\cos\alpha$$

Cor.2. If $e=1$, then $\theta = \alpha$ and $v=u$ i.e., when a perfectly elastic sphere impinges on a fixed smooth plane its velocity is unaltered in magnitude by the impact and the angle of reflections is equal to the angle of incidence.

Cor.3. If $e=0$, then $\cot\theta=0$, or $\theta=90°$ [from (3)] and $v=u\sin\alpha$ [from (4)]. Thus if the sphere and the plane are both inelastic, the sphere moves along the plane with velocity $u\sin\alpha$.

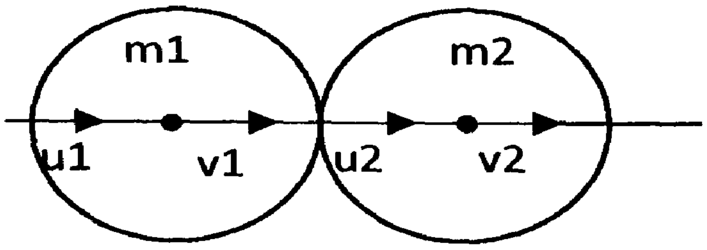
Cor.4. If $\alpha=0$ then $\theta=0$ and $v=e\,u$ from (3) and (4) i.e., if a sphere impinges normally on a horizontal plane, it rebounds vertically with velocity eu.

Cor.5. The change in K.E. of the sphere due to impact on the plane is given by

$$\tfrac{1}{2} m (v^2-u^2) = \tfrac{1}{2} m (v+u)(v-u) = \tfrac{1}{2} I (v+u)$$

Here, $m(v-u) = I$ = Impulse of the force of the sphere on the plane.

1.10 Direct impact of two smooth spheres:



A smooth sphere of mass m1 moving with a velocity u1 impinges on another smooth sphere of mass m2 moving in the same direction with velocity u2. If *e* is the coefficient of restitution between them, find the velocities of the spheres after impact.

Since the spheres are smooth, there is no impulsive force on either along the common tangent. Hence in this direction their velocities after impact are the same as their original velocities i.e., zeroes. Let *v1 and v2* be the velocities of the two spheres along the common normal after impact (Fig).

By the principle of conservation of momentum,

$$m_1v_1+m_2v_2=m_1u_1+m_2u_2 \qquad \text{........ (1)}$$

By Newton's experimental law,

$$v_1 - v_2=-e\ (u_1-u_2) \qquad \text{....... (2)}$$

Multiplying (2) by *m2 and adding to (1),*

$$v_1\ (m_1+m_2) = m_2u_2\,(1+e) + u_1\,(m_1-em_2)$$

$$\therefore \quad v_1 = \frac{m_2u_2\,(1+e) + u_1(m_1-em_2)}{(m_1+m_2)} \qquad \text{...... (3)}$$

Multiplying (2) by m1 and subtracting from (1),

$$v_2\,(m_1+m_2)= m_1u_1(1+e)+u_2(m_2-em_1)$$

$$v_2= \frac{m_1u_1\,(1+e) + u_2(m_2-em_1)}{(m_1+m_2)} \qquad \text{..... (4)}$$

Equation (3) and (4) give the velocities of the two spheres after impact.

Cor.1.The impulse of the blow on the sphere of mass m_1=change of momentum produced in it = $m_1 (v_1-u_1) = v_1 = \frac{m_1+m_2 (1+e) + (u_2 - u_1)}{(m_1+m_2)}$

This is equal and opposite to the impulse of the blow on the sphere of mass m_2.

Cor.2.If $e=1$ and $m_1=m_2$, then, $v_1=u_2$ and $v_2=u_1$. Thus, if two equal perfectly elastic spheres impinge directly, they interchange their velocities.

1.10.1 Loss of K.E. due to direct impact of two smooth spheres

Let m_1, m_2 be the masses, u_1 and u_2, v_1 and v_2 their velocities before and after impact and e the coefficient of restitution. Then, by the principle of conservation of linear momentum,

$$m_1v_1+ m_2v_2= m_1u_1+ m_2u_2 \quad .....(1)$$

By Newton's experimental law,

$$v_1- v_2= -e (u_1- u_2) \quad .... (2)$$

Square both equations multiply the square of the second by m_1m_2 and add the results. Then,

$$(m_1^2+m_1m_2) v_1^2+ (m_2^2+m_1m_2) v_2^2=(m_1u_1+m_2u_2)^2+e^2m_1m_2(u_1-u_2)^2$$

$$(m_1^2+m_1m_2) v_1^2+(m_2^2+m_1m_2)v_2^2=(m_1u_1+m_2u_2)^2+m_1m_2(u_1-u_2)^2$$
$$+ e^2 m_1m_2(u_1-u_2)^2-m_1m_2(u_1-u_2)^2$$

$$\therefore (m_1+m_2)(m_1v_1^2+m_2v_2^2) = (m_1+m_2)(m_1u_1^2+m_2u_2^2)-m_1m_2(u_1-u_2)^2(1-e^2)$$

$$\frac{1}{2}m_1v_1^2+\frac{1}{2}m_2v_2^2=\frac{1}{2} m_1u_1^2+ \frac{1}{2} m_2u_2^2- \frac{1}{2} \frac{m_1m_2}{(m_1+m_2)} (u_1-u_2^2)(1-e^2)$$

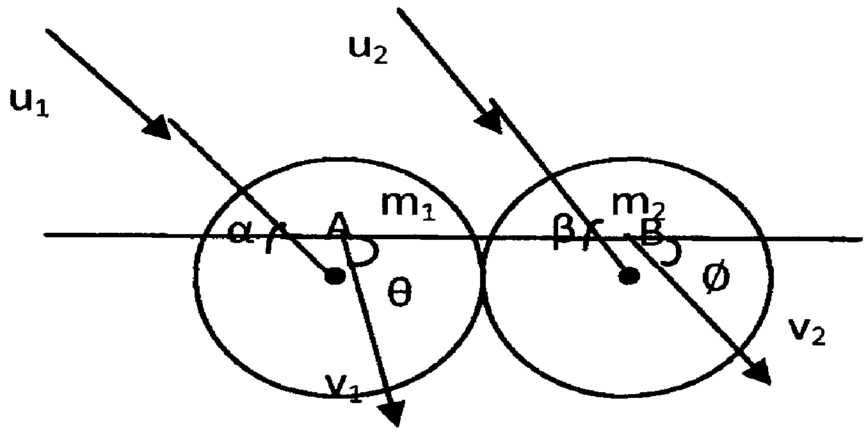
Now $\frac{1}{2}m_1v_1^2+\frac{1}{2}m_2v_2^2=$ *K.E* after impact

$\frac{1}{2}m_1u_1^2+\frac{1}{2}m_2u_2^2=$ *K.E.* before impact

$\therefore$The loss in $K.E = \frac{1}{2} \frac{m_1m_2}{(m_1+m_2)} (u_1-u_2^2) (1-e^2)$

1.11 Oblique impact of two smooth spheres

A smooth sphere of mass m_1 moving with velocity u_1 impinges obliquely on a smooth sphere of mass m_2 moving with velocity u_2. If the directions of motion before impact make angles α and β with the common normal, find the velocities and direction of the spheres after impact.



Let AB be the common normal (Fig). Let v1 and v2 be the velocities of the two spheres after impact making angles θ and $\emptyset$ with the common normal AB. Before impact velocities along the common normal AB are $u_1\cos\alpha$ and $u_2\cos\beta$ and velocities perpendicular to AB are $u_1\sin\alpha$ and $u_2 \sin \beta$. After impact velocities along AB are $v_1 \sin \theta$ and $v_2 \sin \emptyset$.

By the principle of conservation of momentum, the total momentum of the two spheres along the common normal is unaltered by the impact.

$$\therefore m_1v_1\cos\theta + m_2v_2\cos\emptyset = m_1u_1\cos \alpha + m_2u_2\cos\beta \qquad ......(1)$$

By the Newton's experimental law (on relative velocities along the common normal),

$$v_1\cos\theta - v_2\cos\emptyset = -e\,(u_1\cos \alpha - u_2\cos \beta) \qquad .....\ (2)$$

Since there is no force perpendicular to the common normal AB, the velocities of the spheres perpendicular to the common normal AB remain unaltered due to impact. Hence

$$v_1 \sin \theta = u_1 \sin \alpha \qquad ......\ (3)$$

$$v_2 \sin \emptyset = u_2 \sin \beta \qquad ......\ (4)$$

Multiplying (2) by m_2 and adding to (1),

$$(m_1+m_2)\, v_1 cos\theta = (m_1 - em_2)\, u_1\cos \alpha + m_2\,(1+e)\, u_2\cos\beta$$

$$v_1 \cos\theta = \frac{(m_1 - em_2)\, u_1 \cos\alpha + m_2 (1+e)\, u_2 \cos\beta}{(m_1+m_2)} \qquad \ldots\ldots (5)$$

Multiplying (2) by m_1 and subtracting from (1) we get,

$$v_2 \cos\emptyset = \frac{(m_2 - em_1)\, u_2 \cos\beta + m_1 (1+e)\, u_1 \cos\alpha}{(m_1+m_2)} \qquad \ldots\ldots (6)$$

Square (3) and (5) and adding we get v_1^2 and hence we can find v_1. Dividing (3) by (5) we get $\tan\theta$. Similarly, from (4) and (6) we can get v_2 and $\tan\emptyset$. Therefore v_1, v_2, θ and $\emptyset$ are determined uniquely.

Cor.1: The impulse of the blow on the sphere of mass m_1= its change of momentum measured along the common normal = $m_1 v_1 \cos\theta - m_1 u_1 \cos\alpha$

$$= m_1 (v_1 \cos\theta - u_1 \cos\alpha) = \frac{m_1 m_2 (1+e)}{(m_1+m_2)} (u_2 \cos\beta - u_1 \cos\alpha)$$

This is equal and opposite to the impulse on the sphere of mass m_2.

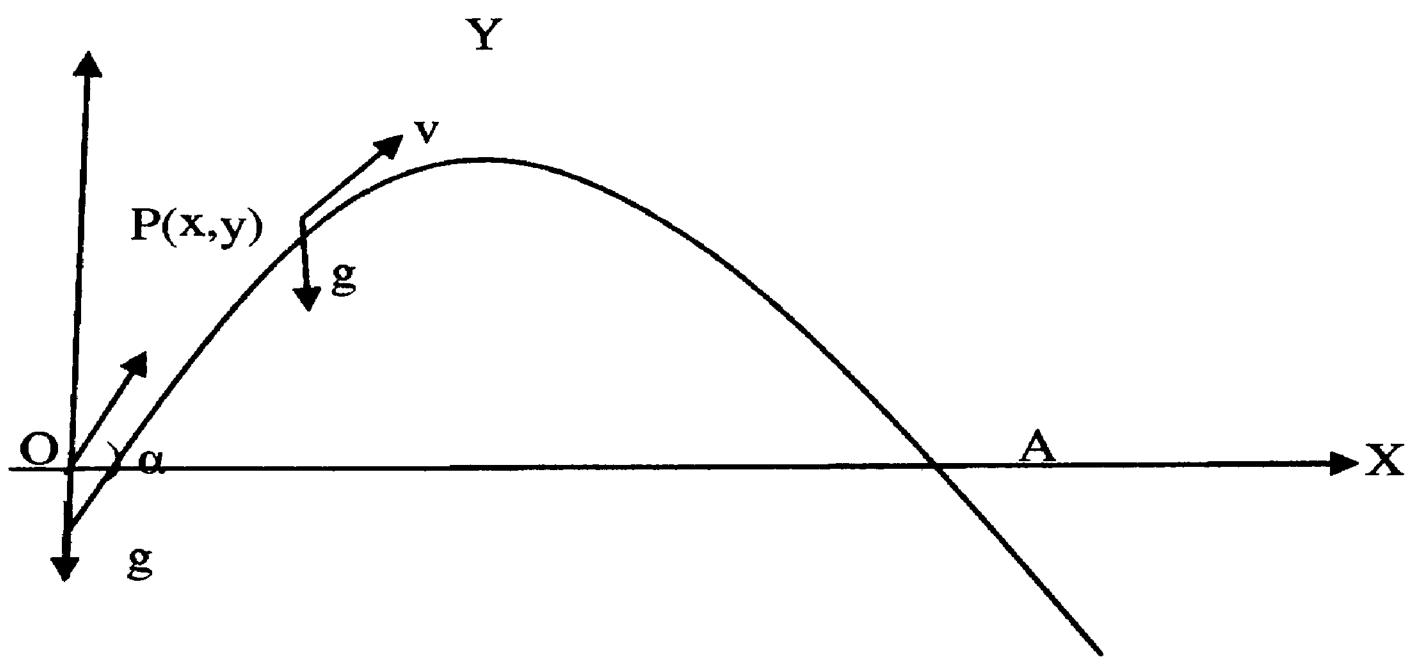
1.11.1 Loss of K.E due to oblique impact

The velocities of the spheres perpendicular to the common normal are unaltered. Therefore, the loss of K.E is the same as in the case of direct impact if we substitute $u_1 \cos\alpha$ and $u_2 \cos\beta$ for u_1 and u_2 respectively.

$$\therefore \text{ the loss in K.E } = \frac{m_1 m_2 (1+e)}{2 (m_1+m_2)} (u_1 \cos\alpha - u_1 \cos\beta)$$

1.12 Projectile motion

Here, we shall consider the motion of a particle projected into air with a given velocity in a particular direction. Such a particle is called a Projectile. We shall assume that the acceleration due to gravity is constant. We shall also neglect the resistance of the air to the motion. The following terms are used in connection with projectiles:



(a) The **angle of projection** is the angle that the direction of projection makes with the horizontal plane through the point of projection.
(b) The **Trajectory** is the path described by the particle.
(c) The **Range** is the distance between the point of projection and the point where the trajectory meets any plane through the point of projection.
(d) The time taken by the particle to describe the horizontal range through **the** point of projection is called the **Time of flight**.

1.12.1 The path of a projectile is a Parabola:

To show that the path of a projectile is a Parabola, we suppose that a particle is projected with velocity 'u' from a point O. OY be an upward vertical through O. Let P (x, y) be the position of the particle at any time t. Since is the velocity of projection of the particle at an angle α with the horizontal, the horizontal component of the initial velocity = u_x=u cosα and the vertical component of the initial velocity u_y=u sin α

Since gravity acts vertically, it has no effect on the velocity of the particle in a horizontal direction.

∴The horizontal distance travelled in time t = x= u t = (u cosα) t .... (1)

Since there is an acceleration –g acting on the particle along the vertical direction,

The vertical distance travelled in time t = y = (u sin α) t-gt^2 ...... (2)

From relation (i), $t=\frac{x}{u\cos\alpha}$

Substituting this value of t in relation (2)

$$y= (u \sin \alpha) \frac{x}{u \cos \alpha} - ½ g (\frac{x}{u \cos \alpha})^2$$

Or $$y = x \tan \alpha - \frac{gx^2}{2u^2 cos^2 \alpha} \quad ..... (3)$$

This equation represents a parabola with its axis vertical.

Note: when y=0, equation (iii) gives α=0, and $x = \frac{2u^2 sin\alpha cos\alpha}{g}$. The zero value of x corresponds to the origin O, which is a point on the path. The other value gives OA, the horizontal range.

1.12.2 Expressions for maximum height, time of flight and range of projectiles.

Consider a particle projected from a point O with a velocity 'u' at an angle α to the horizontal through O. The horizontal component of the velocity of projection is u_x = u cos α and the vertical component of the initial velocity is u_y = u sin α. The horizontal component remains constant throughout the motion. The vertical component is subject to an acceleration –g.

Maximum height

When the particle reaches the highest point, its vertical velocity = 0 and the particle is moving horizontally. In the equation $v^2 = u^2 + 2as$, substituting v = 0,

u =u sin α, a = -g and s = H, we have

$$0 = u^2 \sin^2 \alpha - 2gH$$

Hence, maximum height $H = \frac{u^2 sin^2 \alpha}{2g}$

Time of flight

Let T be the time taken by the particle to describe the arc OA. Then, in this time, the distance described by the particle in the vertical direction is zero.

In the equation $s= ut + \frac{1}{2} at^2$, substituting s=0,

$$0 = (u \sin \alpha) T - \frac{1}{2} aT^2 \qquad \text{or} \qquad T = \frac{2u \sin \alpha}{g}$$

Range: The time of flight = T = $\frac{2u \sin \alpha}{g}$. During the time T the particle has been moving horizontally with uniform velocity u cos α.

The horizontal distance described = (u cos α) T = (u cos α) $\frac{2u \sin \alpha}{g}$

$$= \frac{2u^2 \sin \alpha \cos \alpha}{g} = \frac{u^2 \sin 2\alpha}{g}$$

$$\therefore \text{ The range } R = \frac{u^2 \sin 2\alpha}{g}.$$

1.12.3 Projectile Special case

For the sake of simplicity, let us consider, a particle projected horizontally with a velocity u from the top of a tower. It will move in the horizontal direction with the same horizontal velocity u throughout its motion. Its horizontal motion is represented by x= ut..... (1) where x is the distance travelled in the horizontal direction in time t. The force due to gravity pulls it down in the vertical direction at the same time. The vertical motion is represented by $y= \frac{1}{2} gt^2$

Form equation (1) t= x/u. substituting this value of t in equation (2), $y=\frac{1}{2}g\,\frac{x^2}{u^2}$

Or $\quad x^2 = \frac{2x^2}{g}y.$ This is the equation of a parabola.

1.13 Frictional forces

When we move a book along a table, it moves through a small distance then slows down and finally stops. This indicates that a force is opposing the motion. Whenever one surface moves in contact with another, a force is generated to retard the motion. This force is called **friction**. Friction plays a vital role in our life. For example, man is able to walk on the road because of the friction between his feet and the ground. In machines friction reduces efficiency. Lubrication and ball bearings reduce friction in machines.

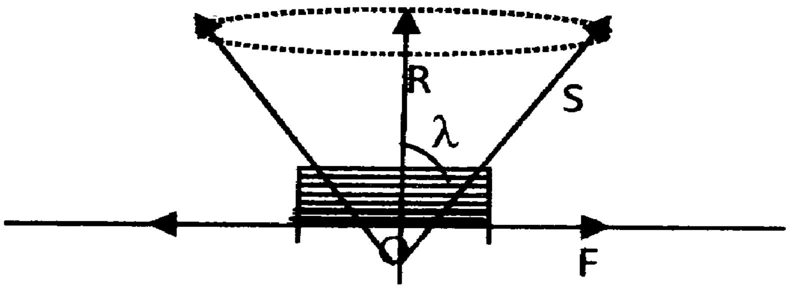
The frictional forces acting between surfaces at rest with respect to each other are called forces of **static friction**. The maximum value of the frictional force between two bodies is called **limiting friction.**

The frictional forces acting between the surfaces in relative motion are called **kinetic or dynamic friction.** The frictional forces acting between two surfaces when one rolls over the other is called **rolling friction.** When a body slides over a surface the frictional force is called **sliding friction.** It is easy to roll a cylindrical body than to slide it. Thus, the rolling friction is less than the sliding friction. We require more force to start motion of a body than to keep it in uniform motion. Thus, static friction is larger than dynamic friction.

Laws of static friction

1. The direction of the frictional force is always opposite to the direction in which the one body tends to slide over another.
2. The magnitude of force of friction is just sufficient to prevent the motion of one body over the other.
3. The frictional force attains a maximum value when one body is just on the point of sliding over the other. The maximum value of the force of friction is called the limiting friction.
4. The magnitude of force of limiting friction bears a constant ratio to the normal reaction between the two bodies. The ratio is called the coefficient of friction and is denoted by the symbol μ. If F is the limiting friction and R is the normal reaction between two bodies, then $\mu = \frac{F}{R}$. The coefficient of friction depends only on the nature of the surfaces in contact.
5. The limiting friction is independent of the surface in contact, provided the normal reaction is unaltered.
6. When a body is in motion, the direction of friction is still opposite to the direction of motion of the body and is independent of the velocity.

Angle of friction



Let F be the force of limiting friction and R, the normal reaction. Let S be the resultant of these two forces. Then the angle between S and R is called the angle of friction. It is denoted by the symbol λ. Then, $\tan \lambda = \frac{F}{R} = \mu$

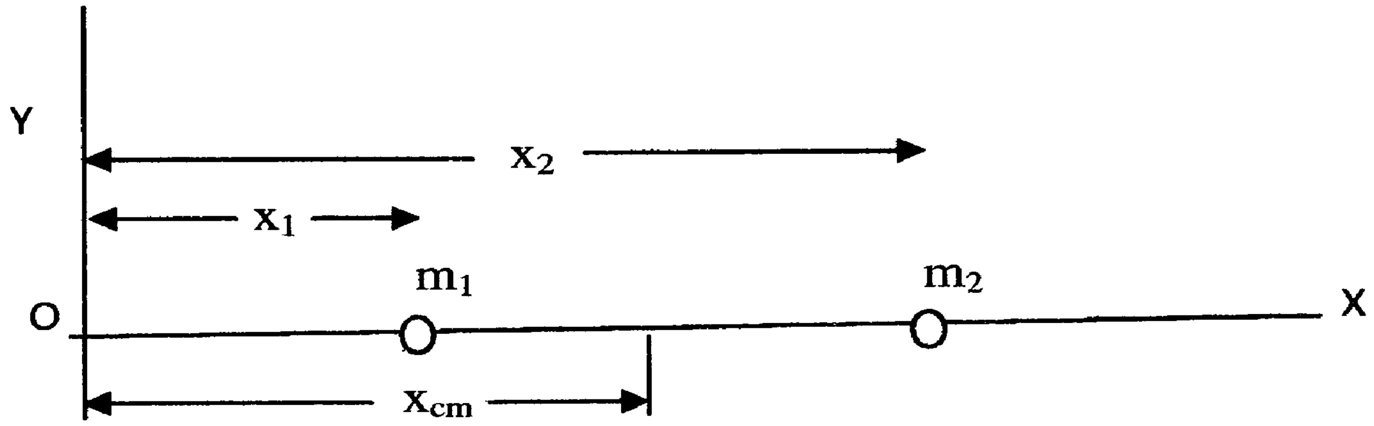
The coefficient of friction = tan (angle of friction)

Cone of friction

Consider a cone with the point of contact of two bodies as the vertex O, the normal reaction R as the axis and the angle of friction λ as semi vertical angle. This imaginary cone is called the cone of friction. The resultant S of the force F and the normal reaction R may lie anywhere on the surface of the cone.

1.4 Centre of mass of a solid object

Consider a system consisting of large number of particles (say n). Let $m_1, m_2 \ldots m_n$ be the masses of the individual particles, then the total mass of the system is $M = m_1 + m_2 + \ldots + m_n$. There is one point in the system which behaves as though the entire mass of the system were concentrated at that point. This point is called **centre of mass** of the system.



Consider a system of two particles of masses m_1 and m_2 situated at distances x_1 and x_2 from the origin O along the x-axis. Then the position of the centre of mass is by definition $x_{cm} = \dfrac{m_1 x_1 + m_2 x_2}{m_1 + m_2}$

If we have a system of n particles of masses $m_1, m_2 \ldots m_n$ situated at distances $x_1, x_2 \ldots x_n$ from the origin then the position of the centre of mass is

$$x_{cm} = \frac{m_1 x_1 + m_2 x_2 + \ldots + m_n x_n}{m_1 + m_2 + \ldots + m_n} = \frac{\sum m_i x_i}{\sum m_i}$$

For a large number of particles lying in a plane, the two co-ordinates of the centre of mass are

$$x_{cm} = \frac{\sum m_i x_i}{\sum m_i} \quad \text{and} \quad y_{cm} = \frac{\sum m_i y_i}{\sum m_i}$$

For a large number of particles distributed in space, the three co-ordinates of the centre of mass are

$$x_{cm} = \frac{\sum m_i x_i}{\sum m_i}, \quad y_{cm} = \frac{\sum m_i y_i}{\sum m_I} \quad \text{and} \quad z_{cm} = \frac{\sum m_i z_i}{\sum m}$$

Consider a system consisting of n particles of masses $m_1, m_2, \ldots, m_n$ with $r_1, r_2 \ldots r_n$ as their position vectors. The position vector r_{cm} of the centre of mass of the system is given by the equation

$$r_{cm} = \frac{m_1 r_1 + m_2 r_2 + \ldots + m_n r_n}{m_1 + m_2 + \ldots + m_n} = \frac{\sum m_i r_i}{\sum m_I} = \frac{\sum m_i r_i}{M}$$

or $$M\, r_{cm} = \sum m_i r_i$$

i.e , the product of the total mass of the system and the position vector of the centre of mass is equal to the sum of the products of the individual mass and their respective position vectors.

A rigid body consisting of a large number of particles compactly paced together. So to find the centre of mass of a solid object, the sign of summation is replaced by that of integration taken over the whole volume of the body. $r_{cm} = \frac{1}{M} \int r dm$

Symmetry considerations are often useful in finding the position of the centre of mass. Thus the centre of mass of homogeneous sphere, cube, circular disc or rectangular plate is at the centre. The centre of mass of cylinder or right circular cone is on the axis of symmetry. The centre of mass of a body may or may not lie in the body itself. For example, the centre of mass of a circular ring is its centre which is not on the material of the ring.

1.5 Conservation of momentum in a system of particles

Suppose we have a system containing n particles. The particles have masses $m_1, m_2 \ldots m_n$ and move with velocities $v_1, v_2 \ldots v_n$. Let M (= $m_1 + m_2 + \ldots + m_n$) be the total mass of the body and V be the velocity of the centre of mass of the system then the momentum of the system is

$$P = p_1 + p_2 + \ldots\ldots + p_n \qquad \ldots\ldots\ldots (1)$$

The total momentum of the system is equal to the product of the total mass of the system and the velocity of its centre of mass.

$$M\,V = m_1v_1 + m_2v_2 + \ldots + m_nv_n \qquad \ldots\ldots\ldots (2)$$

Assuming a constant mass M, the derivative of the momentum is

$$\frac{dP}{dt} = M\frac{dV}{dt} = Ma \qquad \ldots\ldots\ldots (3)$$

where 'a' is the acceleration of the system. According to Newton's second law of motion $\frac{dP}{dt}$ = F, the external force acting on the system of particles. If the velocity of the system is constant then the acceleration becomes zero and hence the force acting on the system is zero.

$$\therefore \quad \frac{dP}{dt} = 0$$

Or $\qquad P = \text{constant.}$

So in systems in which the total mass remains constant, if the net external force acting is zero then total linear momentum P of the system remains constant or conserved.

Exercise I

Section A

1. State the three Newton's laws of motion
2. Define force and give its unit
3. State Newton's second law of motion and obtain the unit of force.
4. What is meant by momentum?
5. Explain the term impulse. What is the relation force and impulse?
6. Distinguish between elastic and inelastic collision.
7. Define Newton's law of impact and coefficient of restitution.
8. State the difference between direct impact and oblique impact.
9. What is a projectile? Define range of a projectile.
10. Define angle of friction and cone of friction.
11. State the laws of friction.
12. What is meant by centre of mass?

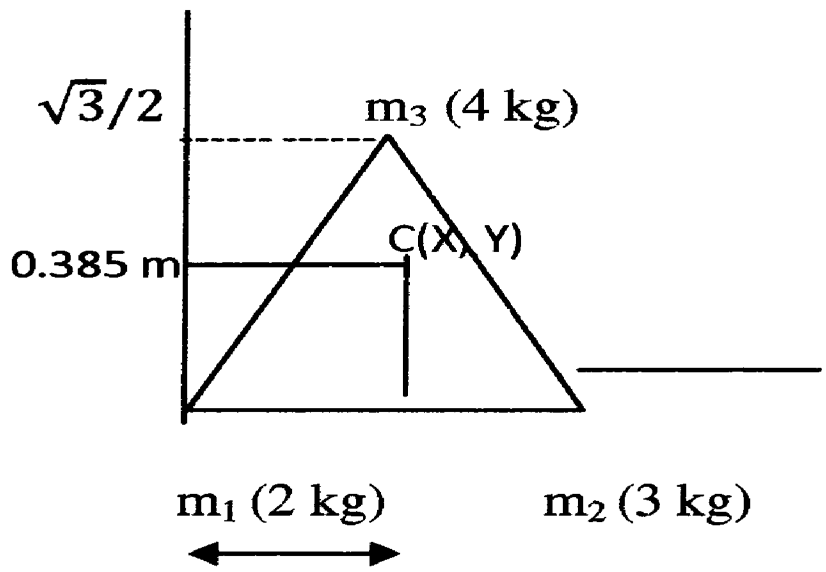
Section B

13. State and explain Newton's laws of motion.
14. Explain the terms 'momentum' and 'impulse'.
15. State and explain the principle of conservation of linear momentum.
16. A smooth sphere has an oblique impact on a fixed smooth plane. Derive expressions for the magnitude and direction of the velocity of the sphere after impact.
17. Discuss the direct impact of two smooth spheres. Derive expressions for their final velocities.
18. A ball of mass 8 kg moving with a velocity of 10 ms^{-1} impinges directly on another of mass 24 g moving at 2 ms^{-1} in the opposite direction. If e =0.5, find the velocities of the balls after impact. (Ans: $v_1 =$ - 3.5 ms^{-1}, $v_2 =$ 2. ms^{-1})
19. Show that the path of a projectile is a parabola.
20. Discuss about the centre of mass of solid object.
21. Show that when the vector sum of the external forces acting upon a system equals to zero, the total linear momentum of the system remains constant.

Section C

22. What is a projectile? Derive general equation for the motion of a projectile. Calculate the expression for (i) maximum height (ii) range and (iii) time of flight.
23. Explain the various types of friction with example. Define (i) Angle of friction (ii) Coefficient of friction and (iii) Cone of friction. State the laws of friction.
24. Derive expressions for the velocities and direction of two smooth spheres after direct impact. Calculate the loss energy due to the direct impact.
25. Discuss the oblique impact of two spheres. Derive expression for the loss of kinetic energy.
26. If an oblique collision occurs between two equal smooth perfectly elastic spheres one of which is initially at rest, show that their path after impact are at right angles to one another.
27. Locate the centre of mass of three particles of mass 2 kg, 3g and 4 kg placed at the three corners of an equilateral triangle of 1 m side.

Solution: Let the triangle lie in the x-y plane with its corner m_1 (2 kg) at the origin and the side m_1 m_2 along the x – axis. Let (X,Y) be the co-ordinates of the centre of mass.



$$X = \frac{\sum m_i r_i}{\sum m} = \frac{2(0) + 3(1) + 4(0.5)}{2+3+4} = 0.56 \text{ m}$$

$$Y = \frac{\sum m_i r_i}{\sum m} = \frac{2(0) + 3(0) + 4(\sqrt{3}/2)}{2+3+4} = 0.385 \text{ m}$$

UNIT II – CIRCULAR MOTION

STRUCTURE

2.1 Introduction

Special cases often dominate our study of physics, and circular motion is certainly no exception. We see circular motion in many instances in the world; a bicycle rider on a circular track, a ball spun around by a string, and the rotation of a spinning wheel are just a few examples. Various planetary models described the motion of planets in circles before any understanding of gravitation. The motion of the moon around the earth is nearly circular. The motions of the planets around the sun are nearly circular. Our sun moves in nearly a circular orbit about the center of our galaxy, 50,000 light years from a massive black hole at the center of the galaxy. We shall describe the kinematics of circular motion, the position, velocity, and acceleration, as a special case of two-dimensional motion. We will see that unlike linear motion, where velocity and acceleration are directed along the line of motion, in circular motion the direction of velocity is always tangent to the circle. This means that as the object moves in a circle, the direction of the velocity is always changing. When we examine this motion, we shall see that the direction of change of the velocity is towards the center of the circle. This means that there is a non-zero component of the acceleration directed radially inward, which is called the centripetal acceleration. If our object is increasing its speed or slowing down, there is also a non-zero tangential acceleration in the direction of motion. But when the object is moving at a constant speed in a circle then only the centripetal acceleration is non-zero.

In all of these instances, when an object is constrained to move in a circle, there must exist a force F acting on the object directed towards the center. In 1666, twenty years before Newton published his Principia, he realized that the moon is always "falling" towards the center of the earth; otherwise, by the First Law, it would continue in some linear trajectory rather than follow a circular orbit. Therefore there must be a centripetal force, a radial force pointing inward, producing this centripetal acceleration.

2.2 Objective

The student should be able to

- Recognize that objects moving in circles have acceleration and explain the cause of this acceleration.
- Describe the magnitude and direction of the acceleration and net force vector of an object moving in a circle at a constant speed.
- Use the concept of inertia to explain the reason that objects moving in circles have a tendency to move tangent to the circle.
- Analyze a physical situation involving circular motion and compare the magnitude of the individual forces which act upon an object.
- Recognize key elements of Newton's law of universal gravitation.
- Identify the variables effecting the force of gravity and predict the effect of alterations in these variables upon the force of gravity.
- Identify the variables effecting the orbital speed of a satellite and discuss the dependence of orbital speed upon these variables.
- Identify the variables effecting the acceleration and net force acting upon an orbiting satellite and discuss the dependence of acceleration and F_{net} upon these variables.

2.3 Uniform circular motion

In physics uniform circular motion describes the motion of a body traversing a circular path at constant speed. The distance of the body from the axis of rotation remains constant at all times. Though the body's speed is constant, its velocity is not constant: velocity, a vector quantity, depends on both the body's speed and its direction of travel. This changing velocity indicates the presence of acceleration; this centripetal acceleration is of constant magnitude and directed at all times towards the axis of rotation. This acceleration is, in turn, produced by a centripetal force which is also constant in magnitude and directed towards the axis of rotation.

In the case of rotation around a fixed axis of a rigid body that is not negligibly small compared to the radius of the path, each particle of the body describes a uniform circular motion with the same angular velocity, but with velocity and acceleration varying with the position with respect to the axis.

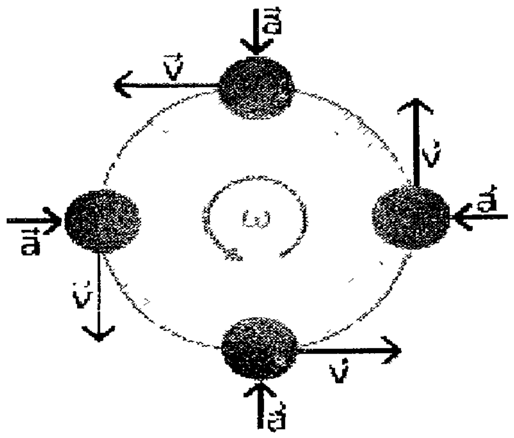


Fig: Velocity **v** and acceleration **a** in uniform circular motion at angular rate ω; the speed is constant, but the velocity is always tangent to the orbit; the acceleration has constant magnitude, but always points toward the center of rotation

Formulas for uniform circular motion

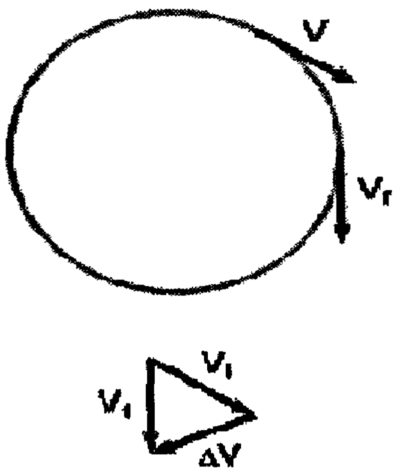
For motion in a circle of radius, the circumference of the circle is $C = 2\pi\ r$. If the period for one rotation is T, the angular rate of rotation, also known as angular velocity ω is: $\omega = \frac{2\pi}{T}$. The unit of angular velocity is radians/sec

The speed of the object traveling the circle is: $v = \frac{2\pi r}{T} = \omega r$

The angle θ swept out in a time t is: $\theta = 2\pi \frac{t}{T} = \omega t$

The acceleration due to change in the direction is: $a = \frac{v^2}{r} = \omega^2 r$

2.4 Dynamics of uniform circular motion



Before discussing the dynamics of uniform circular motion, we must explore its kinematics. Because the direction of a particle moving in a circle changes at a constant rate, it must experience uniform acceleration. But in what direction is the particle accelerated? To find this direction, we need only look at the change in velocity over a short period of time:

The diagram above shows the velocity vector of a particle in uniform circular motion at two instants of time. By vector addition we can see that the change in velocity, Δv , points toward the center of the circle. Since acceleration is the change in velocity over a given period of time, the consequent acceleration points in the same direction. Thus we define centripetal acceleration as an acceleration towards the center of a circular path. All objects in uniform circular motion must experience some form of uniform centripetal acceleration.

We find the magnitude of this acceleration by comparing ratios of velocity and position around the circle. Since the particle is traveling in a circular path, the ratio of the change in velocity to velocity will be the same as the ratio of the change in position to position.

Thus: $\frac{\Delta v}{v} = \frac{\Delta r}{r} = \frac{v\Delta t}{r}$

Rearranging the equation, $\frac{\Delta v}{\Delta t} = \frac{v^2}{r}$

Thus $a = \frac{v^2}{r}$

We now have a definition for both the magnitude and direction of centripetal acceleration: it always points towards the center of the circle, and has a magnitude of v^2/r. Let us examine the equation for the magnitude of centripetal acceleration more practically. Consider a ball on the end of a string, being rotated about an axis. The ball experiences uniform circular motion, and is accelerated by the tension in the string, which always points toward the axis of rotation. The magnitude of the tension of the string (and therefore the acceleration of the ball) varies according to velocity and radius. If the ball is moving at a high velocity, the equation implies, a large amount of tension is required and the ball will experience a large acceleration. If the radius is very small, the equation shows, the ball will also be accelerated more rapidly.

Centripetal force

Centripetal force is the force that causes centripetal acceleration. By using Newton's Second Law in conjunction with the equation for centripetal acceleration, we can easily generate an expression for centripetal force.

$$F_c = ma = \frac{mv_\circ^2}{r}.$$

Remember also that force and acceleration will always point in the same direction. Centripetal force therefore points toward the center of the circle.

There are many physical examples of centripetal force. In the case of a car moving around a curve, the centripetal force is provided by the *static* frictional force of the tires of the car on the road. Even though the car is moving, the force is actually perpendicular to its motion, and is a static frictional force. In the case of an airplane turning in the air, the centripetal force is given by the lift provided by its banked wings. Finally, in the case of a planet rotating around the sun, the centripetal force is given by the gravitational attraction between the two bodies.

With knowledge of physical forces such as tension, gravity and friction, centripetal force becomes merely an extension of Newton's Laws. It is special, however, because it is uniquely defined by the velocity and radius of the uniform circular motion. All of Newton's Laws still apply, free body diagrams are still a valid method for solving problems, and forces can still be resolved into components. Thus the most important thing to remember regarding uniform circular motion is that it is merely a subset of the larger topic of dynamics

2.5 Moment of inertia of a Circular Lamina or Disc

(i) About an axis through its centre and perpendicular to its plane.

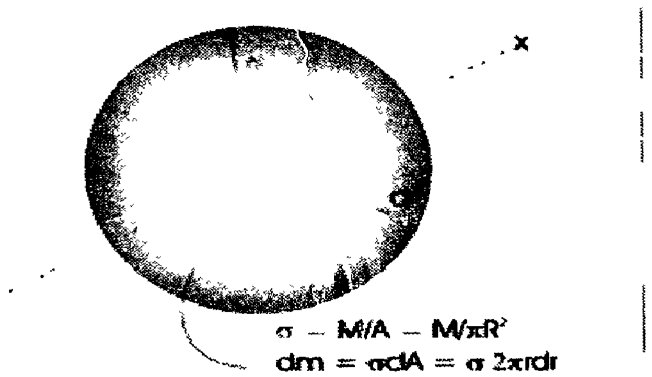
Let M be the mass of the disc and R, its radius. Then, since the area of the disc is πR^2, its mass per unit area will be $M/\pi R^2$.

Consider an element of the disc at a distant r from O, i.e., a ring of radius r and of width dr, (Figure). Its area is clearly equal to its circumference, multiplied by its width.

i.e., area of the ring $= 2\pi r \times dr$

mass of the ring $= \frac{M}{\pi R^2} 2\pi r\, dr$

$= \frac{2M r\, dr}{R^2}.$



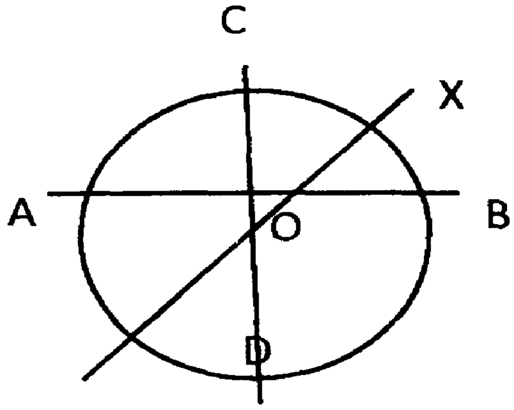
Hence, moment of inertia of this ring about an axis through O and perpendicular to its plane $= \frac{2Mr.dr}{R^2} r^2 = \frac{2Mr^3 dr}{R^2}$.

Since the whole disc may be supposed to be made up of such like concentric rings of radii ranging from O to R, we can get the moment of inertia / of the disc by integrating the above expression for the moment of inertia of the ring, for the limits x=0, and x=R.

M.I. of the disc $= \int_0^R \frac{2M}{R^2}. r^3.dr = \frac{2M}{R^2}. \frac{R^4}{4}$.

Or, $I = \frac{1}{2} MR^2$

(ii) About its diameter



Let AB and CD be two perpendiculars of a circular disc of radius R and mass M, (Figure). Since the moment of inertia of the disc about one diameter is the same as about any other diameter, the moment of inertia about the diameter AB is equal to the moment of inertia about the diameter CD, perpendicular to AB. Let it be I. Now, we have, by the principle of perpendicular axes,

M.I. of the disc about AB+M.I about CD = M.I. about an axis through O and perpendicular to its plane.

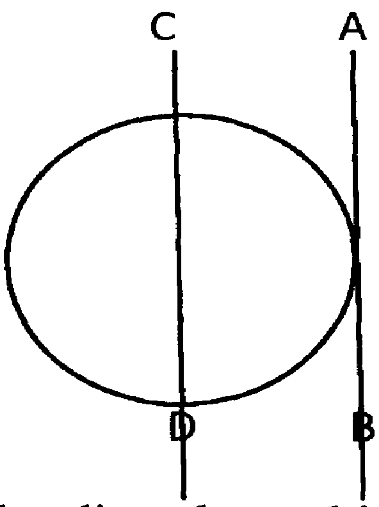
Or, $I + I = \frac{1}{2} MR^2$

Or $2I = \frac{1}{2} MR^2$.

Or, $I = \frac{1}{4} MR^2$

(ii) About a tangent to the disc in its own plane

Let AB be tangent to the circular disc of radius R and mass M, about which its moment of inertia is to be determined, (Figure). Let CD be a diameter of the disc, parallel to the tangent AB.



The moment of inertia of the disc about this diameter is, clearly equal to $MR^2/4$. So that, by the principle of parallel axes, we have

M.I. of the disc about AB = M.I. of the disc about CD+MR^2.

Or, $$I = (\frac{1}{4} MR^2) + MR^2 = \frac{5}{4} MR^2$$

(iv) About a tangent to the disc and perpendicular to its plane

This tangent will obviously be parallel to the axis through the centre of the disc and perpendicular to its plane, the distance between the two being equal to the radius of the disc. Hence, by the principle of parallel axes, we have

M.I. about the tangent = M.I. about the perpendicular axis+MR^2.

Or, $$I = \frac{1}{2} MR^2 + MR^2$$

$$I = \frac{3}{2} MR^2$$

2.6 Moment of Inertia of a Solid Sphere

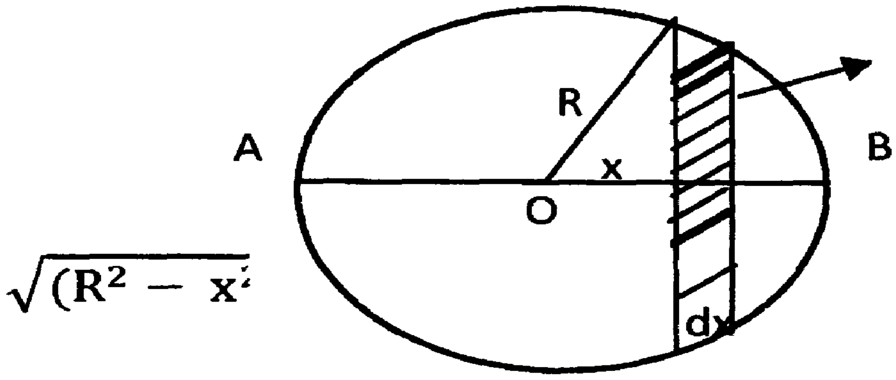
(i) About its diameter

Let Figure represent a section of the sphere through its centre O. Let mass of the sphere be M and its radius, R.

Then, its volume $= \frac{4}{3}\pi R^3$

Mass per unit volume $= \frac{M}{\frac{4}{3}\pi R^3}$

$= \frac{3M}{4\pi R^3}$



Consider a thin circular slice of the sphere at a distance x from the centre O and of thickness dx. This slice is obviously a disc of radius $\sqrt{R^2 - x^2}$, and of thickness dx.

∴ Surface area of the slice $= \pi\ (\sqrt{(R^2 - x^2)}\ )^2$

$= \pi(R^2 - x^2)$

Volume of the slice = area × thickness

$= \pi(R^2 - x^2).dx.$

Mass of the slice = its volume × mass per unit volume of the sphere

$= \pi(R^2 - x^2).dx \times \frac{3M}{4\pi R^3} = \frac{3M.(R^2 - x^2)}{4R^3}.dx.$

Now, the moment of inertia of this disc about AB (an axis passing through its centre and perpendicular to its plane)

= its mass x $(\text{radius})^2 / 2$.

∴ moment of inertia of the disc AB $= \frac{3M.(R^2 - x^2)}{4R^3}\ dx\ \frac{(R^2 - x^2)}{2}$

$$= \frac{3M(R^2 - x^2)^2}{8R^3}\ dx \quad \ldots (1)$$

∴moment of inertia I of the sphere about the diameter AB is equal to twice the integral of expression (1) between the limits x = 0 and x = R.

$$I = 2\int_0^R \frac{3M(R^2 - x^2)^2}{8R^3}dx = \frac{2\times 3M}{8R3}\int_0^R (R^2 - x^2)^2 dx.$$

$$= \frac{3M}{4R^3}\int_0^R (R^4 - 2R^2x^2 + x^4)dx$$

$$= \frac{3M}{4R^3}\left[R^4x - 2R^2\frac{x^3}{3} + \frac{x^5}{5}\right]_0^R$$

$$= \frac{3M}{4R^3}\left(R^5 - \frac{2}{3}R^5 + \frac{1}{5}R^5\right) = \frac{3M}{4R^3}\left[\frac{15R^5 - 10R^5 + 3R^5}{15}\right]$$

or, $I = \frac{3M}{4R^3}\times\frac{8R^5}{15} = \frac{2}{5}MR^2.$

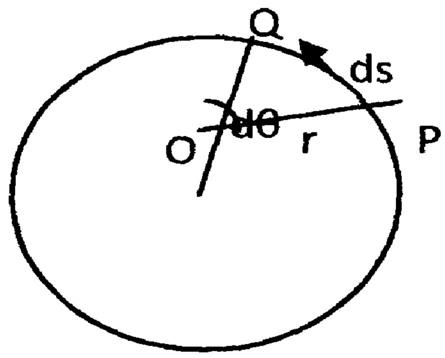
(ii) About a tangent

A tangent, drawn to the sphere at any point, will obviously be parallel to one its diameters and at a distance from it equal to R, the radius of the sphere. Therefore, in accordance with the principle of parallel axes, we have

M.I. of the sphere about a tangent = M.I. about the diameter + MR^2.

$$I = \frac{2}{5}MR^2 + MR^2 = \frac{7}{5}MR^2$$

2.7 Angular momentum, Angular velocity and Torque



Angular velocity

Suppose a particle moves from a point P to Q in a small interval of time dt. Let dθ be the angle described by the radius vector during this time interval. The rate of change of angular displacement is called **angular velocity** and is denoted by ω.

$$\omega = \frac{d\theta}{dt} \quad \text{............ (1)}$$

From the figure, ds = r dθ, therefore, $\frac{ds}{dt} = r\frac{d\theta}{dt}$

$$v = r\omega \quad \text{............. (2)}$$

That is, Linear velocity = radius x angular velocity.

Angular momentum

In the case of linear motion, the momentum of a body is the product of its mass and velocity. In the case of rotational motion, the product of the moment of inertia and the angular velocity is the **angular momentum** of a rotating body.

If I is the moment of inertia and ω, the angular velocity of the body about the axis of rotation, then, angular momentum

$$L = I\,\omega \qquad \ldots\ldots\ldots\ldots (3)$$

Angular acceleration

If the angular velocity increases with time then the particle is said to have angular acceleration α. The **angular acceleration** is defined as the rate of change of angular velocity.

$$\alpha = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2} \qquad \ldots\ldots\ldots\ldots (4)$$

Differentiating equation (2) w. r. t time, we have, $\frac{dv}{dt} = r\frac{d\omega}{dt}$

$$v = r\omega \qquad \ldots\ldots\ldots\ldots (5)$$

That is, linear acceleration = radius x angular acceleration.

Torque

We require a force to act on a body in order to move through a distance. The expression for force is F=mass x acceleration. Similarly in order to make a body rotate about an axis we require a **torque**. The torque is give by the expression,

Torque = mass x angular acceleration

$$\tau = I \times \alpha. \qquad \ldots\ldots\ldots\ldots (6)$$

2.8 Relation between angular momentum and torque

The expression for the Angular momentum is given by $L = I \times \omega$ (Where I is the Moment of inertia of the rigid body and ω is its angular velocity.)

The expression for the Torque $\tau = I \times. \alpha$ (where I is the Moment of inertia and α is angular acceleration.)

Therefore $\tau = I \times \frac{d\omega}{dt}$ [since $\alpha = d\omega/dt$]

$$= \frac{d}{dt}(I\,\omega)$$

$$= \frac{d}{dt}(L)$$

$$\tau = \frac{dL}{dt}$$

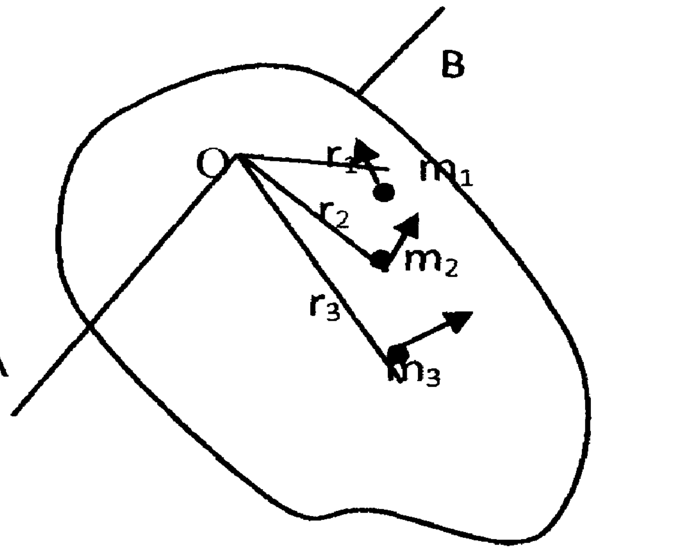
Therefore the Torque is nothing but the rate of change of Angular momentum.

2.9 Kinetic Energy of Rotation

Kinetic energy of a body about an axis through its centre of mass

Suppose we have a body of mass M rotating about an axis AB, passing through its centre of mass O, (Fig). It, obviously, possesses kinetic energy due to its motion; this energy of the body is called its energy of rotation, because it is due to its motion of rotation.

Imagine the body to be divided up into a large number of small particles, of masses m_1, m_2, m_3, etc., at distances r_1, r_2, r_3 etc. respectively from the axis AB.



Then, we have

linear velocity of $m_1 = r_1\ \omega = v_1$,

linear velocity of $m_2 = r_2\ \omega = v_2$,

and linear velocity of $m_3 = r_3\ \omega = v_3$ and so on.

Kinetic energy of the mass $m_1 = \frac{1}{2} m_1\ v_1^2$;

Kinetic energy of the mass $m_2 = \frac{1}{2} m_2\ v_2^2$; and Kinetic energy of the mass $m_3 = \frac{1}{2} m_3\ v_3^2$ and soon.

Or, total K.E.of the body $= \frac{1}{2} m_1\ r_1^2\ \omega^2 + \frac{1}{2} m_2\ r_2^2\ \omega^2 + \frac{1}{2} m_3\ r_3^2\ \omega^2 + \ldots\ldots$

$$= \frac{1}{2}\omega^2[m_1\ r_1^2 + m_2\ r_2^2 + m_3\ r_3^2 + \ldots\ldots\ldots]$$

$$= \frac{1}{2}\omega^2\ \Sigma\ mr^2 = \frac{1}{2}\omega^2 MK^2 \qquad [\because \Sigma\ mr^2 = MK^2.$$

Since $MK^2 = I$, the M.I. of the body about the axis AB,

$$\text{K.E. of the body} = \frac{1}{2} I\ \omega^2,$$

Now, if $\omega = 1$, then, obviously, K.E. of the body $= \frac{1}{2}$ I.

Or, $\qquad I = 2$ K.E.

This gives us another definition of moment of inertia, viz. that the moment of inertia of a body, rotating with unit angular velocity, is equal to twice its kinetic energy of rotation.

2.10 Work energy theorem

Suppose a force F acts on a body at a distance r from the axis about which the body can rotate (Figure). The body rotates through a small angle $d\theta$. The work done by the force is

$$dw = F\ ds = Frd\theta = \tau\ d\ \theta$$

Work done by a torque = torque × angular displacement.

But $\qquad \tau = I\ \alpha$

$$dw = \tau\ d\ \theta = I\ \alpha\ d\ \theta = I\frac{d\omega}{dt} d\ \theta = I\frac{d\theta}{dt} d\omega = I\ \omega\ d\ \omega.$$

Total work done when the angular velocity changes from ω_1 to ω_2

Is

$$W = \int_{\omega_1}^{\omega_2} I\omega d\omega = I\left[\frac{\omega^2}{2}\right]_{\omega_1}^{\omega_2}$$

∴ $$W = \frac{1}{2} I\,\omega_2^{\,2} - \frac{1}{2} I\,\omega_1^{\,2}$$

Since the work done by the torque on a rotating body is equal to the increase in the kinetic energy of the body, this is termed as Work Energy theorem.

2.11 Conservation of angular momentum

The law of conservation of momentum states that when no external torque acts on an object or a closed system of objects, no change of angular momentum can occur. Hence, the angular momentum before an event involving only internal torques or no torques is equal to the angular momentum after the event. The time derivative of angular momentum is called torque:

$$\tau = \frac{dL}{dt} = \frac{dr}{dt} \times p + r \times \frac{dp}{dt} = 0 + r \times F = r \times F$$

(The cross-product of velocity and momentum is zero, because these vectors are parallel.) So requiring the system to be "closed" here is mathematically equivalent to zero external torque acting on the system:

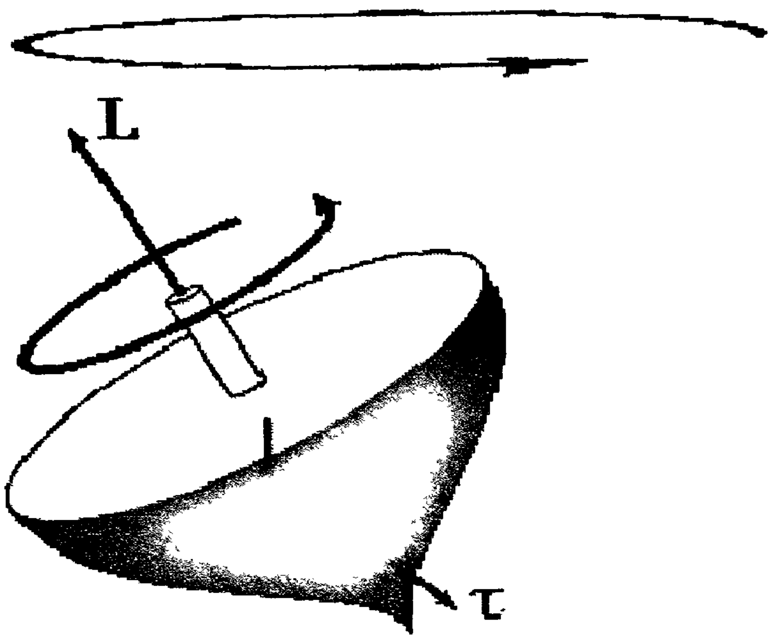
L_{system} = constant (i.e) $\sum \tau_{ext} = 0$

where τ_{ext} is any torque applied to the system of particles. It is assumed that internal interaction forces obey Newton's third law of motion in its strong form, that is, that the forces between particles are equal and opposite and act along the line between the particles.

In orbits, the angular momentum is distributed between the spin of the planet itself and the angular momentum of its orbit:

$L_{total} = L_{spin} + L_{orbit}$

If a planet is found to rotate slower than expected, then astronomers suspect that the planet is accompanied by a satellite, because the total angular momentum is shared between the planet and its satellite in order to be conserved. The conservation of angular momentum is used extensively in analyzing what is called *central force motion.* If the net force on a body is directed always toward some fixed point, the *center*, then there is no torque on the body with respect to the center, and so the angular momentum of the body about the center is constant. Constant angular momentum is extremely useful when dealing with the orbits of planets and satellites, and also when analyzing the Bohr model of the atom.



The conservation of angular momentum explains the angular acceleration of an ice skater as she brings her arms and legs close to the vertical axis of rotation. By bringing part of mass of her body closer to the axis she decreases her body's moment of inertia. Because angular momentum is constant in the absence of external torques, the angular velocity (rotational speed) of the skater has to increase.

The same phenomenon results in extremely fast spin of compact stars when they are formed out of much larger and slower rotating stars (indeed, decreasing the size of object 10^4 times results in increase of its angular velocity by the factor 10^8).

The conservation of angular momentum in Earth–Moon system results in the transfer of angular momentum from Earth to Moon (due to tidal torque the Moon exerts on the Earth). This in turn results in the slowing down of the rotation rate of Earth (at about 42 ns/day[1]), and in gradual increase of the radius of Moon's orbit (at ~4.5 cm/year rate).

2.12 Work done by constant force

Work is said to be done when external force acting on a particle displaces it.

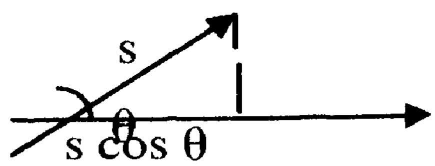
Definition of work

It is defined as the product of force and displacement in the direction of force.

Let F be the constant force and 's' the displacement of the particle (figure). θ is the angle between them. Then, the work done by the force F is

$$W = F.s = Fs \cos \theta.$$

Work = (force) × (displacement along the direction of force).



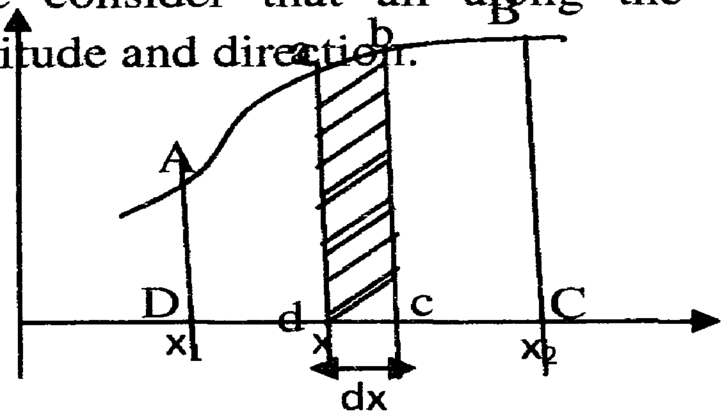
The unit of work is joule.

One joule is defined as the work done by a force of 1N which acting on a particle displaces the particle by 1 m along the direction of force.
Dimensional formula for work is $[ML^2 T^{-2}]$.

2.13 Work done by a varying Force

If the force is varying, then work done may be found graphically. Let a graph be plotted between force F versus displacement x (figure).

The curve AB in the graph shows the variation of the magnitude of the force with the change in position of the particle. We want to calculate the work done as body moves from position A to B. We consider the entire displacement as made up of a large number of infinitesimal displacements. In figure, one such infinitesimal displacement from point a to b is shown. We consider that all along the displacement ab, the force is constant both in magnitude and direction.



Work done by force during displacement from x to x + dx is

dW = F dx = area of rectangular strip abcd.

∴Net work done for displacement from x_1 to x_2,

$$W = \Sigma F\,dx = \int_{x_1}^{x_2} F\,dx = \text{Area between F – x curve and x – axis} = \text{area ABCD}.$$

2.14 Work and energy in rotational motion

Suppose a force F acts on a body at a distance r from the axis about which the body can rotate (Figure). The body rotates through a small angle dθ. The work done by the force is

$$dw = F\,ds = Frd\theta = \tau\,d\theta$$

Work done by a torque = torque × angular displacement.

But $\tau = I\,\alpha$

$$dw = \tau\,d\theta = I\,\alpha\,d\theta = I\frac{d\omega}{dt}d\theta = I\frac{d\theta}{dt}d\omega = I\,\omega\,d\omega.$$

Total work done when the angular velocity changes from ω_1 to ω_2 is

$$W = \int_{\omega_1}^{\omega_2} I\omega d\omega = I\left[\frac{\omega^2}{2}\right]_{\omega_1}^{\omega_2}$$

$$\therefore \quad W = \frac{1}{2}I\,\omega_2^{\,2} - \frac{1}{2}I\,\omega_1^{\,2}$$

Thus the work done by the torque on a rotating body is equal to the increase in the kinetic energy of the body.

2.15 Acceleration of a body rolling down an inclined plane

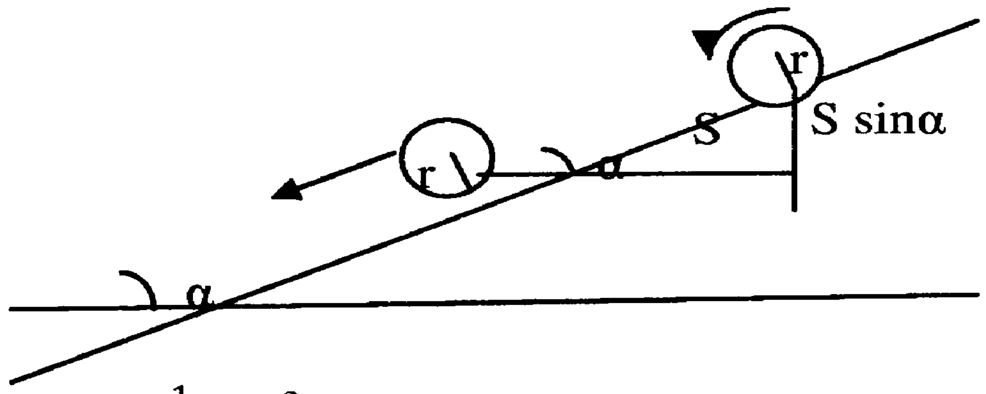
Let a body like a disc or sphere etc. (i.e., a body having circular symmetry, of mass M roll freely down an inclined plane, of inclination α to the horizontal, (Figure). The plane is supposed to be rough enough, so that there may be no slipping, and hence no work done by friction.

Then, if v be the velocity acquired by the body after traversing a distance S along the plane, we have vertical distance through which it has descended = S $\sin\alpha$.

$\therefore$ P.E. lost by the body = Mg S $\sin\alpha$. ....(1)

This must be equal to the K.E gained by the body.

Now, K.E of rotation of the body = $\frac{1}{2}$ Iω^2 (where ω is its angular velocity about a perpendicular axis through its centre of mass.)



Its K.E. of translation = $\frac{1}{2}$ Mv^2, (because its center of mass has a linear velocity v.)

$\therefore$ total K.E. gained by the body = $\frac{1}{2}$ Iω^2 + $\frac{1}{2}$ Mv^2 ....(2)

$$= \frac{1}{2} Mv^2 [\frac{k^2}{R^2} + 1]$$

Since gain in K.E. of the body is equal to the loss in its P.E, we have

$$\frac{1}{2} Mv^2 [\frac{k^2}{R^2} + 1] = Mg.\ S \sin\alpha.$$

Or, $\quad Mv^2 [\frac{k^2}{R^2} + 1] = 2Mg.\sin\alpha.S.$

Or, $\quad v^2 [\frac{k^2+R^2}{R^2}] = 2g.\sin\alpha.S,$

Whence, $v^2 = 2[\frac{R^2}{k^2+R^2}]\ g \sin\alpha\ S.$

Comparing this with the kinematic relation, $v^2 = 2aS$, for a body starting from rest, we have acceleration of the body down the plane,

$$a = [\frac{R^2}{k^2+R^2}]\ g \sin\alpha.$$

Or, the acceleration is proportional to $[\frac{R^2}{k^2+R^2}]$ for a given angle of inclination α.

This shows that

(i) The greater the value of K, as compared with R, the smaller the acceleration of the body coming down the plane and, therefore, the greater the time it takes in rolling down along it and vice versa;

(ii) The acceleration and, therefore, the time of descent, is independent of the mass of the body.

Let us consider some particular cases.

(i) Solid Sphere. Here $k^2 = \frac{2}{5} R^2$ and, therefore, acceleration down the plane,

$$a = (R^2 / K^2 + R^2)\ g.\ \sin\alpha = [R^2 / (\tfrac{2}{5}R^2 + R^2)]\ g \sin\alpha = \tfrac{5}{7}\ g \sin\alpha.$$

(ii) Spherical Shell. In this case, $K^2 = \frac{2}{3} R^2$ and therefore ,

$$a = [R^2 / (\tfrac{2}{3} R^2 + R^2)] g \sin\alpha = \tfrac{3}{5}\ g \sin\alpha.$$

(iii) Disc or cylinder. In this case, K^2 (about its own axis) = $R^2 / 2$, and, therefore, $a = [R^2 / (\frac{R^2}{2} + R^2)]\ g \sin\alpha.$

(iv) Ring or Hoop. Here, obviously, $K^2 = R^2$ and , therefore,

$$a = [R^2 / (R^2 + R^2)] g \sin\alpha.$$

: follows at once from the above that if a solid sphere, a spherical shell, a disc (or ylinder) and a hoop, all of the same mass, be allowed to roll down an inclined lane (without slipping) they will come down the plane in this very order, viz, the olid sphere first, then, the spherical shell, followed next by the disc (or the ylinder) and lastly the hoop. And, since K^2 for a hollow sphere about its diameter ; greater than that for a solid sphere of the same mass and radius, they can be istinguished from each other by allowing them to roll down the plane. Obviously, ıe solid sphere will roll down faster than the hollow one.

EXERCISE II

PART A

1. Define angular momentum.
2. Define angular velocity.
3. Write the relation between angular momentum and torque.
4. Write down the law of conservation of angular momentum.
5. Define Work.
6. Write about the relation between work and energy.
7. Define Torque.
8. What is centripetal force? Write the expression for it.
9. Write the expression for the moment of inertia of a solid sphere about its tangent.
10. Define work in rotational motion.

PART B

1. Explain the dynamics of uniform circular motion.
2. Derive an expression for the moment of inertia of a circular disc about an axis passing through its centre and perpendicular to its plane.
3. Obtain an expression for the kinetic energy of rotation.
4. Explain Work Energy theorem.
5. Describe about the work done by a variable force.
6. Explain about work and kinetic energy in rotational motion.

PART C

1. Derive an expression for the moment of inertia of a solid sphere about its diameter.
2. Obtain an expression for the acceleration of a body rolling down an inclined plane and also discuss about various special cases.

UNIT III GRAVITATION

Structure

3.1 Introduction

3.2 Objectives

3.3 Newton's law of gravitation

3.4 Kepler's law of planetary motion

3.5 Mass and density of earth

3.6 Determination of G – Boys' Method

3.7 Variation of g with altitude

3.8 Variation of g with depth

3.9 Variation of g with latitude

3.10 Gravitation field and potential

3.11 Gravitational potential due to a spherical Shell

3.12 Gravitational field due to a spherical Shell

3.13 Satellites

- 3.13.1 Orbital velocity
- 3.13.2 Escape velocity
- 3.13.3 Stationary satellite

3.14 Jet plane

3.15 Rocket – Principle, Theory

3.16 Velocity of rocket at any instant

3.17 Rocket propulsion systems

3.18 Specific impulse

3.19 Multistage Rocket

3.20 Shape of the rocket

3.21 Earthquake

3.22 Seismograph

3.23 Modern applications of seismology.

3.1 Introduction:

We always observe that an object dropped form a height falls towards the earth. It is a matter of common observation that, leaves of the tree, fruit from the tree, body just dropped from the top of the tower etc., all reach the ground. It is said that Newton was sitting under the tree, an apple fell on him. The fall of the apple made Newton start thinking. He proposed the idea that the apple is attracted by the earth with a force called gravitational force. According to Newton third law of motion the apple also attracts the earth Newton concluded that all object in the universe attract each other. This type of interaction between the object is called gravitation.

3.2 Objectives

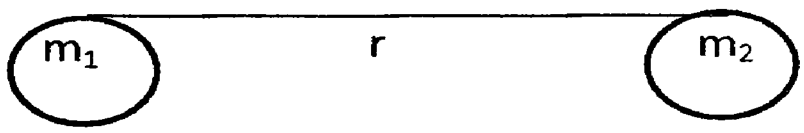
After completing this lesson, you are able to

- recognize that all free-falling bodies accelerate at the same rate due to gravity, regardless of their mass
- describe the difference between g, the acceleration due to gravity, and G, the Universal Gravitational Constant, and also describe gravity as an attractive force among all objects
- compare and describe the gravitational forces between two objects in terms of their masses and the distance between them
- describe weight in terms of the force of a planet's
- explain how an object such as a space station maintains its orbital motion
- describe satellite motion and make calculations related to satellite motion
- make use of the Law of Conservation of Energy regarding orbital motion
- explain the difference between orbital velocity and escape velocity.

3.3 Newton's law of gravitation

Statement: Every particle of matter in the universe attracts every other particle with a force which is directly proportional to the product of their masses and inversely proportional to the square of the distance between them.

Explanation: Let m_1 and m_2 be the masses of two particle situated at a distance r apart (Fig.).



The force of attraction between them is given by

$$F \propto \frac{m_1 m_2}{r^2}$$

Or

$$F = \frac{G m_1 m_2}{r^2}$$

Here, G is a universal constant, called the Universal gravitational constant.

Definition of G: If $m_1=m_2=1kg$ and r=1 m then, F=G.

Thus, the Gravitational constant is defined as the force of attraction between two unit masses of matter unit distance apart.

The unit of G is N m^2 kg^{-2}. The value of G is $6.67 \times 10^{-11} Nm^2kg^{-2}$.

The dimensions of G are $[M^{-1}L^3T^{-2}]$.

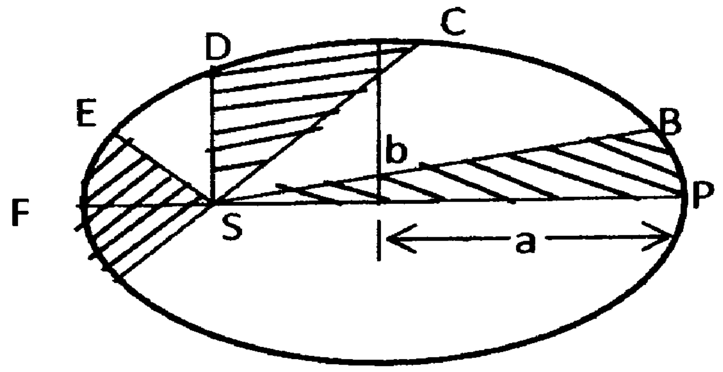
3.4 Kepler's law of planetary motion

I LAW: Each planet revolves around the sun in elliptical orbits, with the sun at one focus of the ellipse.

Explanation: In following Fig. *P* is the planet as *S* is the Sun. *a* and *b* are semi – major and semi – minor axis respectively.

II LAW: The radius vector, drawn from the sun to a planet, sweeps out equal areas in equal interval of times.

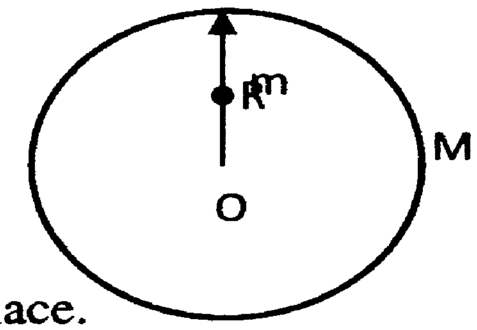
Explanation: The areal velocity of the radius vector is constant (*dA/dt*=constant). If the planet moves from A to B, C to D, E to F in equal time interval Δt, then the areas ASB, CSD and ESF are equal. When the planet is nearest to sun, its velocity is maximum. When the planet is farthest from the sun, its velocity is minimum.



III LAW: The square of the period of revolution of the planet around the sun is proportional to the cube of the semi-major axis of the ellipse ($T^2 \propto a^3$).

Explanation: From this law it follows that larger is the distance of the planet from the sun, larger will be the period of revolution. The planet Venus is nearest to sun. Its period of revolution is only 88 days. The planet Pluto is farthest to sun. Its period is 248 years.

3.5 Mass and density of earth



Consider a body of mass m placed on the surface of earth. Let M be the mass of the earth and g be the acceleration due to gravity at hat place.

The force of attraction of the earth on the body = mg .... (1)

Gravitational force of attraction between the body and the earth = $\frac{GMm}{R^2}$ .... (2)

Comparing equations (1) and (2) $\frac{GMm}{R^2} = mg$

∴The mass of the earth is $M = \frac{gR^2}{G}$ .... (3)

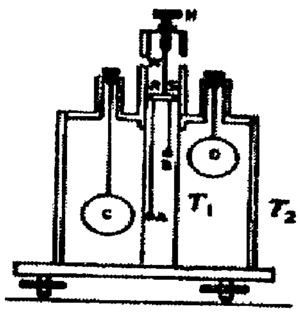
The density of the earth = $\frac{\text{mass of the earth}}{\text{volume}} = \frac{M}{V}$

$$= \frac{gR^2/G}{\frac{4}{3}\pi R^3}$$

∴Density of the earth $\rho = \frac{3g}{4\pi RG}$ .... (4)

3.6 Determination of G – Boys' Method

Apparatus: (i) A small mirror strip RS is suspended for torsion head H by means of a quartz fiber W (Fig). From the two ends of the mirror, two identical gold spheres *A* and *B* are suspended. The two spheres are arranged to be at different levels. The arrangement is enclosed I a fixed inner tube T_1.



(iii) In the outer co-axial tube T_2, two large lead spheres C and D are suspended. The center of C is in level with that of A, the center of D is in level with that of B and the distance AC = BD.

Working: The outer tube is rotated so that the larger spheres are on the opposite sides of the smaller spheres. Gravitational attraction between the two pairs of masses produces a torque o the mirror strip *RS*. Due to this torque, the mirror is deflected through an angle θ. The deflection θ is measured by a scale ad telescope arrangement.

Calculation:

Let M = mass of the lead sphere,

m = mass of the gold sphere,

d = distance between the two sphere.

Force of attraction between spheres *A* and *c* is

$$F = \frac{GMm}{d^2}.$$

Force of attraction between sphere *B* and *D* is

$$F = \frac{GMm}{d^2}.$$

Hence two equal unlike parallel forces act at *R* and *S*

Let l = length of the mirror strip *RS*.

The deflecting torque $= \frac{GMm}{d^2} \times l$

A restoring torque is set up in the suspension fibre.

Let c be the torque per unit twist. Then, for a deflection θ, the restoring torque on the fiber = $c\,\theta$.

When the system is in equilibrium, the two torques balance each other.

$$\frac{GMm}{d^2} \times l = c\,\theta.$$

$$G = \frac{c\theta d^2}{mM \times l} \qquad \text{........ (1)}$$

Using the arrangement of the quartz fiber and the mirror strip with gold spheres as a torsion pendulum the period T is found.

Then $$T = 2\pi\sqrt{\frac{I}{C}}. \text{ .... (1)}$$

Here, I = moment of inertia of the suspended system.

From this c can be calculated.

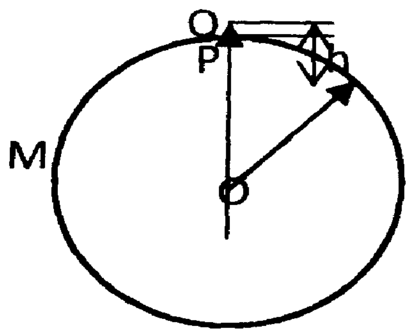
Substituting for c in Eq. (1), G is calculated.

Merits of Boys' Method

1. The use of quartz fiber makes the apparatus very sensitive and accurate.
2. By the scale and telescope arrangement, very small deflections can be measured accurately.
3. The two pairs of spheres are suspended at different levels.
   So cross – attraction between the pairs of spheres is eliminated.

3. 7 Variation of g with altitude

Let p be a point of the surface of the earth and Q another point at an altitude h (Fig). Mass of the earth is M and radius of the earth is R. Let g be the acceleration due to gravity on the surface of the earth.



The force experienced by a body of mass m at p

$$= mg = \frac{GMm}{R^2} \qquad \text{....... (1)}$$

The force experienced by a body of mass m at Q

$$= mg' = \frac{GMm}{(R+h)^2} \qquad \text{........ (2)}$$

Here, g ' *is* the acceleration due to gravity at an altitude h.

Dividing (2) by (1),

$$\frac{g}{g} = \frac{R^2}{(R+h)^2} = \frac{R^2}{R^2\left[1+\left(\frac{h}{R}\right)\right]^2} = \left(1+\frac{h}{R}\right)^{-2}$$

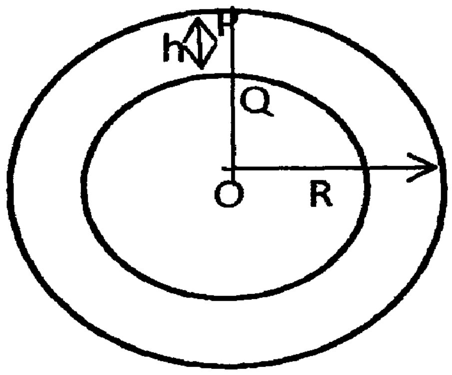
$= (1 - \frac{2h}{R})$ [Neglecting higher powers of $\frac{h}{R}$]

Or $g' = g(1 - \frac{2h}{R})$. This shows that the acceleration due to gravity decreases with increase in altitude.

3.8 Variation of g with depth

Let g and g' be the values of acceleration due to gravity at p and Q respectively (Fig). At P, the whole mass of the earth attracts the body.

$\therefore mg = \frac{GMm}{R^2}$ .... (1)



Here, m =mass of the body.

M = mass of the earth and

R = Radius of the earth

At Q, the body is attracted by the mass of the earth of radius (R-h).

$\therefore mg' = \frac{GM\ m}{(R-h)^2}$ .... (2)

Here, $M = \frac{4}{3}\pi R^3 \rho$ and $m' = \frac{4}{3}\pi(R-h)^3\rho$.

Here, ρ is the mean density of the earth.

Dividing (2) by (1),

$$\frac{g'}{g} = \frac{m'}{M}\frac{R^2}{(R-h)^2}$$

$$= \frac{\frac{4}{3}\pi(R-h)^3\rho}{\frac{4}{3}\pi R^3\rho} \times \frac{R^2}{(R-h)^2}$$

$$= \frac{(R-h)}{R} = (1 - \frac{h}{R})$$

$$\therefore g' = g\left(1 - \frac{h}{R}\right).$$

Therefore, the accelerations due to gravity decrease with increase of depth.

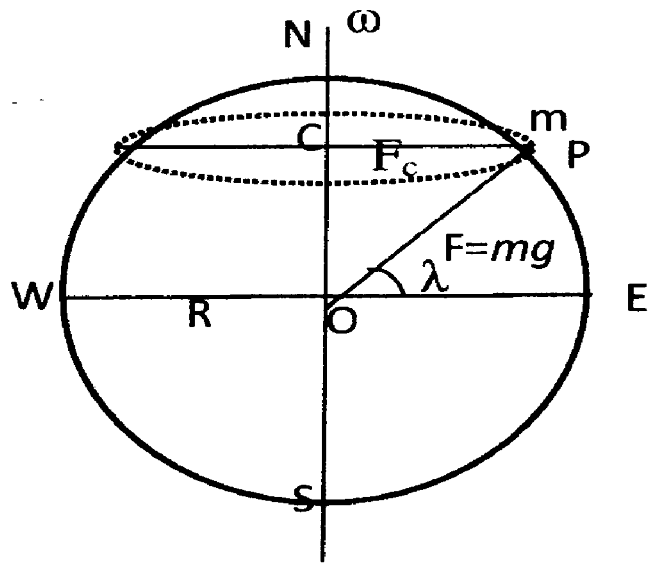
3.9 Variation of g with latitude

The earth is a sphere of radius R rotating with angular velocity ω about an axis passing through its poles (N, S) and center O. Consider a body of mass m at P whose latitude is λ (Fig).

The body situated at P moves in a circular path of radius.

$R = r \cos\lambda$ with angular speed ω. The necessary centripetal F_c is provided by a component of gravitational attraction force between the body and the earth.

The centripetal force $F_c = m\,(CP)\,\omega^2 = mr\,\omega^2 = mR(\cos\lambda)\omega 2$



The effective force along center of earth O

$$=(\text{Force of gravity F}) - \text{component of centripetal force } F_c \text{ along } PO$$

$$= mg - F_c \cos\lambda$$

$$= mg - [mR(\cos\lambda)\,\omega^2] \cos\lambda$$

$$= mg - mR\,\omega^2 \cos^2\lambda$$

Let g be the effective value of g due to earth's rotation. Then,

$$mg' = mg - mR\,\omega^2 \cos^2\lambda$$

$$g' = g - R\,\omega^2 \cos^2\lambda \qquad \ldots\ldots. (1).$$

Or $$g' = g\left(1 - \frac{R\omega^2 \cos\lambda}{g}\right)$$

The equation shows that the value of g decreases due to earth's rotation.

Case (i) At the poles $\lambda = 90°$; $\cos\lambda = 0$

$$\therefore g' = g$$

Case (ii) At the equator, $\lambda = 0°$; $\cos\lambda = 1$

$$g' = g(1 - \frac{R\omega^2}{g})$$

So the value of acceleration due to gravity is maximum at the poles.

3.10 Gravitation field and potential

Gravitational field

The space around a body (of mass m) within which the gravitational force of attraction is perceived is called gravitational field.

The gravitation field intensity (E) at a point is the force experienced by a unit mass at that point.

Explanation:

Consider the body of mass M. let P be a point at a distance r from A. Suppose a particle of mass m kg is placed at P, the force experienced by the particle at P is

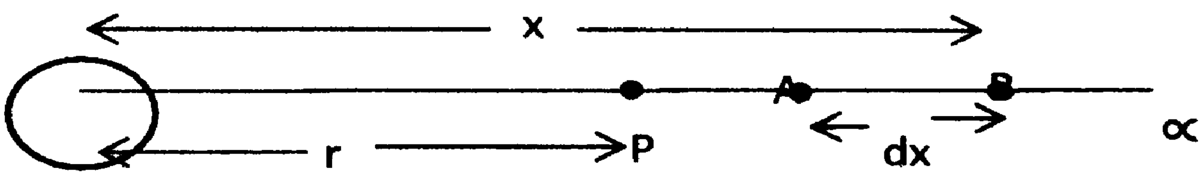
$$F = \frac{GMm}{r^2} \quad .... (1) \quad \text{Newton's law of gravitation}$$

If m=1 kg then the force experienced by a particle of mass 1 kg (unit mass) at P is the gravitational field intensity.

$$\therefore E = \frac{GM}{r^2} \quad .... (2)$$

Comparing (1) and (2) $\quad E = \frac{F}{m} \quad .... (3)$

Gravitational Potential:



Gravitational Potential (V) at a point is defined as the amount of work done in moving a unit mass from the point to infinity against the gravitational force of attraction.

Consider a body of mass M. P is a point at a distance r from the body.

Consider a point B at a distance x from the particle. The gravitational field (force experienced by a unit mass) at $A = \frac{GM}{x^2}$ .... (1)

If the unit mass is displaced from B to A through a distance dx, the work done is

$$dv = -\frac{GM}{x^2}dx \qquad \text{........ (2) [w.d =Force × displacement]}$$

This is the gravitational potential difference between A and B.

The total work done in moving the unit mass from P to ∝ is the gravitational potential at P. It is obtained by integrating dv between the limits r and α.

∴The gravitational potential $V = \int_r^\alpha dv$

$$V = \int_r^\alpha -\frac{GM}{x^2}dx = GM\left[\frac{1}{x}\right]_r^\alpha = GM\left[\frac{1}{\alpha} - \frac{1}{r}\right]$$

$$V = -\frac{GM}{r}$$

Relation between gravitational field and potential

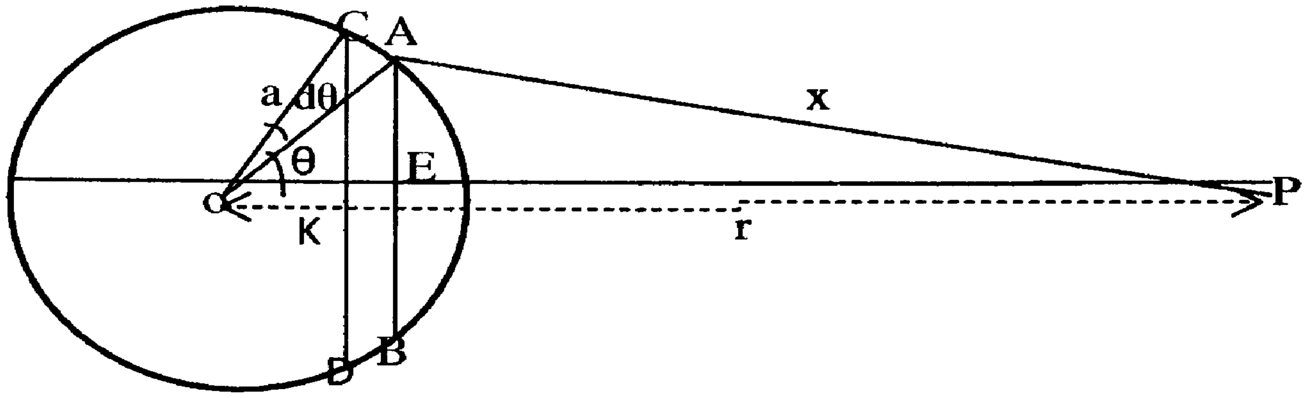
The gravitational field or intensity is also defined as the negative gradient of gravitational potential.

i.e. $$E = -\frac{dv}{dr}$$

On the other words, the gravitational field is the space rate of change of gravitational potential. Here dv is the small charge in gravitational potential for a small distance dr.

3.11 Gravitational potential due to a spherical Shell

(i) Point outside the shell



Consider a point *P* outside the spherical shell at a distance *r* from its center *O* (Fig). Let *a* be the radius of the shell, ρ the mass per unit area of the surface of the shell and *M* its total mass. Join *OP* and let *O=r*. Consider a thin slice of the shell contained between two planes *AB* and *CD* drawn close to each other at right angles to *OP*. Join *O* and A, *O* and C, and A and *P*.

Let ∠AOP =θ, and ∠AOC=dθ.

Now, AE=Radius of the slice = a sin θ.

Circumference of the slice = 2π×AE= 2π a sin θ.

Width of the slice CA= $ad\theta$.

Hence surface area of the slice =2πa sin θ × a dθ

$$= 2\pi a^2 \sin\theta\, d\theta.$$

∴ Mass of the slice = $2\pi a^2 \rho \sin\theta\, d\theta$.

Let *PA* = *x*. Every point on the slice may be taken to be practically equidistant from *P*.

∴ Potential at P due to the ring = $dV = \frac{-G.2\pi a^2 \rho \sin\theta\, d\theta}{x}$ .... (i)

To find the value of x, consider the triangle *OAP*.

$$X^2 = a^2 + r^2 - 2ar\cos\theta.$$

Differentiating 2x dx 2aar sin θ dθ [Since a and r are constants]

Or $x = \frac{ar\sin\theta\, d\theta}{dx}$

Substituting this value of x in (i),

$$dV = \frac{-G.2\pi a^2 \rho \sin\theta \, d\theta . dx}{ar \sin\theta \, d\theta} = \frac{-2\pi a\rho \, G}{r} dx.$$

If the entire shell is split up into slice of this kind, the value of PA will vary from (r-a) to (r+a).

Hence the potential at P due to the entire shell = $V = \int_{r-a}^{r+a} \frac{-2\pi a\rho \, G}{r} dx.$

$$= \frac{-2\pi a\rho \, G}{r} [x]_{r-a}^{r+a} = \frac{-2\pi a\rho \, G}{r} 2a = -4\pi a^2 \rho \frac{G}{r}$$

Now $4\pi a^2 \rho$ = mass of the whole shell.

$$\therefore V = -\frac{GM}{r}$$

This potential is the same as due to a mass *M* at *O*. Hence, the mass of the shell behaves as though it were concentrated at its centre.

(ii) Point on the surface of the shell: Let us consider a point which lies on the surface of the shell itself. The limits for the value of x will be 0 and 2a. Hence,

Potential at a point on the surface of the shell = $V = \int_0^{2a} \frac{-2\pi a\rho \, G}{r} dx$

$$= \frac{-2\pi a\rho \, G}{r} [x]_0^{2a} = \frac{-4\pi a^2 \rho \, G}{r} = \frac{-GM}{r} = \frac{-GM}{a}. \qquad \therefore (r=a).$$

(iv) Point inside the shell:

Let the point *P* be situated at K inside the shell, such that OK=r. The limits for the value of x will be (a-r) and (a+r).

$\therefore$ Potential at a point (K) inside the shell $V = \int_{a-r}^{a+r} \frac{-2\pi a\rho \, G}{r} dx$

$$= \frac{-2\pi a\rho \, G}{r} [x]_{a-r}^{a+r} = -4\pi a\rho \, G$$

Multiplying and dividing by a, $V = \frac{-4\pi a^2 \rho \, G}{a} = \frac{-GM}{a}.$

Hence the potential at all points inside a spherical shell is the same and is equal to the value of the gravitational potential on the surface.

3.12 Gravitational field due to a spherical Shell

The intensity of the gravitational field p is given by $F = \frac{-dv}{dr}$. The gravitational field F in different cases is obtained using this relation.

(i)At a point on the shell:

$$V = \frac{-GM}{r}$$

$$\therefore F = \frac{-dv}{dr} = -\frac{d}{dr}\left[\frac{-GM}{r}\right] = \frac{-GM}{r^2} \qquad \ldots\ldots (i)$$

The negative sign indicates that the force is towards the centre O.

(v) **At a point on the outer surface of the shell:**

Putting r=a in the expression (i), we get the intensity of the gravitational field at a point on the surface of the shell.

$$F = \frac{-GM}{r^2}$$

(vi) At a point inside the shell:

Potential $V = \frac{-GM}{a} = A$ constant, $\therefore F = \frac{-dv}{dr} = 0$

Hence, there is no gravitational field inside a spherical shell.

3.13 Satellite

A satellite is a body which revolves around one of the planets in its own orbit. For example, Moon is a natural satellite of our earth. Artificial satellites can be launched from the Earth's surface to circle to earth.

3.13.1 Orbital velocity

The velocity which an object must acquire to circle the earth in a circular path of radius r is called its orbital velocity.

Let us now calculate the velocity with which the body (of mass m) should be projected form the earth's surface so as to revolve around it i.e., to become its satellite. Let it be V_o. Let the distance of the satellite from the centre of the earth (which is also the radius of the orbit of the satellite) be r. Then, the centrifugal force on the body, tending to take it away from the surface of the earth is =

$\frac{mV_{\circ}^2}{r}$. The gravitational force acting on the body must just counter balance the centrifugal force.

The gravitational force $= \frac{GMm}{r^2}$ (where M=mass of the earth)

$$\therefore \frac{mV^2}{r} = \frac{GMm}{r^2}$$

$$V_o^2 = \frac{GM}{r}$$

$$V_{o\circ} = \sqrt{\frac{GM}{r}}$$

But $mg = \frac{GMm}{r^2}$ or $g = \frac{GM}{r^2}$ $\therefore V_o = \sqrt{gr}$

Thus, orbital velocity (velocity of projection) of a body to become a satellite of the earth is $V_o = \sqrt{gr}$

3.13.2 Escape velocity

The velocity with which a body should be projected to enable it to escape from the gravitational pull of the earth is called the escape velocity.

When a body is projected upwards from the surface of the earth, it rises up to a certain height and on account of the gravitational pull of the earth's surface. The height to which it rises depends on the initial velocity with which it is projected upwards. The grater velocity of projection, the greater is the height. If the body can be given a velocity which can take it beyond the gravitational field of the earth, it will never come back and escape into space. Therefore, this velocity of the body is called the escape velocity.

Expression for escape velocity

Suppose a body of mass m is situated at a height x from the centre of the earth.

The force of attraction on the body = $\frac{GMm}{x^2}$ where M= mass of the earth. If this body is moved upwards through a distance dx,

The work done $= \frac{GMm}{x^2}\, dx$

Therefore the total work done in moving the body from the surface of the earth to infinity = $\int_R^\infty \frac{GMm}{x^2}$ dx (where R= Radius of the earth) = $\frac{GMm}{R}$.

If the body is projected upwards with a velocity V_e, then

The initial K.E. of the body = $\frac{1}{2}mV_e^2$

If this K.E. is equal to the work that has to done by the body during its escape, the body can escape from the gravitational pull of the earth.

Hence, $\frac{1}{2}mV_e^2 = \frac{GMm}{R}$

$$V_e^2 = \frac{2GM}{R}$$

$$V_e = \sqrt{\frac{2GM}{R}}.$$

But $\frac{GM}{R^2} = R$ ∴ $\quad \therefore V_e = \sqrt{2gR}.$

Substituting the values, g=9.8 m s^{-2}; R=64×10^6m, we have $V_e=11.2 \times 10^3$ ms^{-1}.

Or $\quad V_e$=11.2 Kms^{-1}

3.13.3 Stationary Satellite

Consider a satellite in a circular orbit round the earth. It is coplanar with the equator of the earth. It period of rotation is the same as that of the earth. This satellite appears to be stationary to the observer on the earth. Such satellites are used widely for television and radio broadcasting.

The angular velocity of this satellite is given by $\omega = \frac{2\pi}{T}$,

Here T=24 hours = 86400s

$$\therefore \omega = \frac{2\pi}{86400} = 7.3 \times 10^{-5} \text{ radians/sec}$$

The distance of such a satellite from the centre of the earth is given by

$$\omega = \sqrt{\frac{GM}{r^3}}$$

$$r = \left(\frac{GM}{\omega^2}\right)^{1/3} = \left[\frac{6.67\times10^{-11}\times5.98\times10^{24}}{(7.3\times10^{-5})^2}\right]^{1/3}$$

r=4.2×10^7m

This distance is about 6.6 times the radius of the earth. The distance of such a satellite from the surface of the earth is

$h = r - R = 42\times10^6 - 6.4\times10^6$

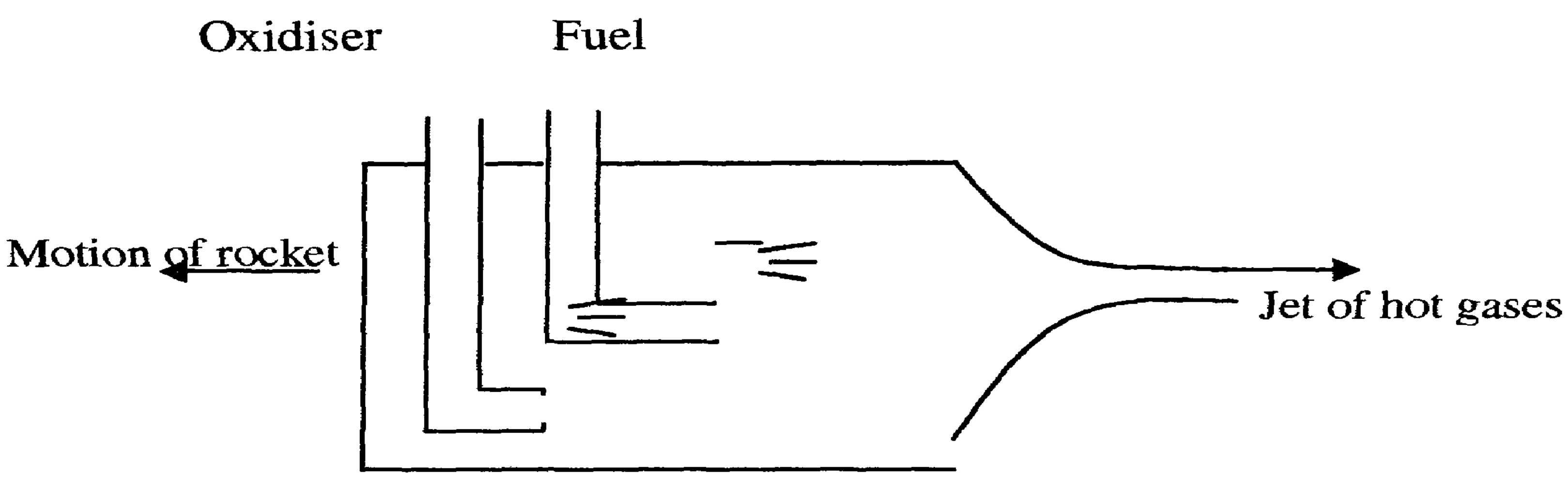
$h = 35.6\times10^6 m = 3.56\times10^7 m$

3.14 Jet Plane

The principle of working of a jet plane is similar to that of a rocket. A jet plane is fitted with a diesel engine. The air is sucked in from the front and the fuel is sprayed into it. This mixture is compressed to a high pressure in the piston chamber and ignition takes place. The burnt gases escape with a high velocity from the rear and impart an equal and opposite momentum to the jet plane in accordance with Newton's third law of motion. These planes can fly at high speeds. If the speed is greater than the velocity of sound, they are called supersonic jets.

The difference between a rocket and jet plane is that, in the case of a rocket the fuel burns quickly and imparts large thrust. In the case o f a plane the consumption of the fuel is continuous.

3.15 Rocket Motion



Principle

In a rocket, the fuel is burnt in a combustion chamber. The large quantity of the heat of combustion produced inside the chamber. As a result, the rocket in the form of a high velocity stream called the JET (Fig). In consequence, the rocket is propelled forward (i.e., opposite to the direction of the jet), since the momentum lost by the jet of fuel gases must be equal to the momentum gained by the rocket.

Theory: Let v be the velocity of the rocket relative to the earth and v' be the exhaust velocity of the gases relative to the earth. Then, the exhaust velocity of the gases relative to the rocket $v_c = v' - v$. v_c *and* v are in opposite directions and v_c is usually constant, let m be the mass of the rocket, including its fuel, and v the velocity of the rocket at any time t.

The momentum of the system at time t= p = mv.

If Δm is the mass of the expelled gases during a very small time interval dt, the mass of the rocket becomes $(m\text{-}\Delta m)$ and the velocity of the rocket increases to v+dv.

The momentum at time t+dt=P'

P' =momentum of rocket + momentum of expelled gases

$= (m-\Delta m)(v+dv)+\Delta m\times v'$

Let us put $\Delta m=-dm$

Then$P'=(m+dm)(v+dv)-v'dm$

$=mv+m\,dv+ vdm-v'dm$(neglecting $dm \times dv$)

$=mv+m\,dv-(v'-v)dm$

$=mv+m\,dv-v_e dm$ (since $v'-v=v_e$).

∴The change in the momentum in the time dt = $dp=P'-P=m\,dv-v_e dm$

Rate of change of momentum of the system=$\frac{dp}{dt} = m\frac{dv}{dt} - v_e\frac{dm}{dt}$

By Newton's second law, this must be equal to external force (F) acting on the rocket. Hence the equation of motion for the rocket is

$$F=m\frac{dv}{dt} - V_e\frac{dm}{dt} \qquad \text{.........} \quad (1)$$

The second term in equation (1) $-V_e\frac{dm}{dt}$ is called the thrust of the rocket sine it is equal to the force due to the escape of the hot gases through the orifice at its tail end.

$$\therefore \text{ Thrust on the rocket } = F = -V_e\frac{dm}{dt} \quad \text{.............} \quad (2)$$

For a rocket, the rocket designer's aim is to make the thrust as large as possible Inspection of equation (2) shows that this requires that the rocket ejects as much mass per unit time as possible and that the exhaust velocity of the gases relative to rocket be as high as possible.

3.16 Velocity of the Rocket at any Instant

The equation of motion for rocket is,

$$F = m\frac{dv}{dt} - v_e\frac{dm}{dt}$$

To solve this equation, we assume that v_e is constant. Also, neglecting air resistance and variation of gravity with altitude, we may write F=mg.

$$\therefore mg = m\frac{dv}{dt} - v_e\frac{dm}{dt}$$

Or $\quad \frac{dv}{dt} - \frac{ve}{m}\frac{dm}{dt} = g \qquad \text{.......... (3)}$

To simplify, consider that the motion is vertical. Then v is directed upward and v_e *and g* downward. Equation (3) can be written as,

$$\frac{dv}{dt} + \frac{ve}{m}\frac{dm}{dt} = -g$$

Multiplying by dt, dv + $\frac{ve}{m}$dm=-g dt.

Integrating from the beginning of the motion (t=0), when the velocity is v' and the mass is m' upto an arbitrary time t, we get

$$\int_{v_\circ}^{v} dv + v_\circ \int_{m_\circ}^{m} \frac{dm}{m} = -g\int_0^t dt$$

Then $v - v_o + v_e \log_e \frac{m}{m_o} = -gt$

Or $\quad v = v_o + v_e \log_e \frac{m_o}{m} - gt \qquad \text{........... (4)}$

3.17 Rocket propulsion

The history of space exploration is also the history of rocket development. The various stages of development in rocket engines culminated in landing man on Moon. Some of the very basic concepts of a rocket engine are given below:

Rocket Propulsion Systems

At present chemical-propellant rockets are employed. In these rockets, hot gases under high pressure are produced in a combustion chamber and acquire their velocity in a nozzle. Rockers of this class can be subdivided into solid propellant, liquid propellant, and hybrid rockets. (1) In the case of a solid-propellant rocket, the propellant consists of the combustibles and an agent supplying the oxygen for its combustion. The propellant is introduced into the combustion chamber, where it burns. In so doing, it produces a hot high pressure gas which is discharged through a nozzle and thus produces the thrust that propels the rocket. The major advantage of the solid-propellant rocket is its relative simplicity in design. (2) In liquid-propellant rockets, the liquid combustibles are contained in tanks and fed into the combustion chamber through an injector head by a propellant supply system. Most liquid-propellant rockets use two combustibles (bipropellant system) such as liquid oxygen (the oxidizer) and kerosene (the fuel). (3) In hybrid rockets, a liquid oxygen supplying agent is used in conjunction with a solid combustible. Chemical-propellant rockets are characterized by high thrust for short-durations. Therefore, they are very suitable for getting large payloads lunched from the ground. For forward propulsion, the ratio of the total thrust developed by the rocket to the total weight should be greater than one.

3.18 Specific impulse

Another important parameter associated with the effectiveness of a rocket is its specific impulse (I_{sp}). It is given by the total thrust divided by the weight of the propellant consumed per second, $I_{sp} = \frac{F}{d\omega/dt}$

Since F and ω are measured in the same units (newton) specific impulse is in seconds. By Newton's law.

$$\frac{d\omega}{dt} = \frac{dm}{dt} \times g$$

Thrust on the rocket = F = $-V_e\frac{dm}{dt}$

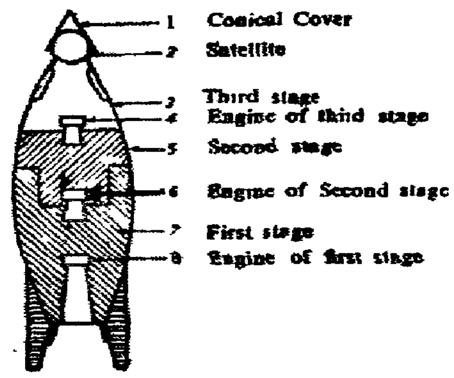
$$\therefore I_{sp} = -\frac{Ve\frac{dm}{dt}}{\frac{dm}{dt}\times g} = -\frac{Ve}{g}$$

The value of g is assumed to be a constant and equal to $9.8ms^{-2}$, wherever the rocket is located. So specific impulse can also be found by dividing V_e (the exhaust velocity of the gases relative to the rocket) by the constant g.

3.19 Multistage Rocket

At the present stage of rocket development no single stage rocket (which has only one engine) can achieve the escape velocity (11.2 km s^{-1})or the orbital velocity near the surface of the earth (8 km s^{-1}). Therefore we have to use a multistage i.e., a two or three stage rocket to achieve the purpose. In the year 1957, Russian scientists sent the artificial satellite Sputnik into the orbit by the three stage rocket.

The multistage rocket is just a combination of rockets with the rear part of one inside the nozzle of the other as shown in Fig 19. The first stage is the largest in dimensions and in weight. The last stage is the smallest and the lightest. When one stage operates, the successive stages are the payload for it. When the rocket is sent in space the first stage rocket is used first and when its fuel is all burnt up, it gets detached and is discarded. Now, the second stage rocket begins to work and it produces further acceleration. The second stage is discarded when its fuel is burnt up and the third stage rocket takes over and so on. Thus the velocity goes on increasing at each stage by the same amount as it does in a single stage rocket.



The final stage, usually the smallest, carries the payload. Its final velocity is the sum of the final velocities attained by all the rocket stages. A typical example of a giant multistage rocket is the Saturn V, which took America's first astronauts to the moon. It stood 263 ft. on the launching pad and weighed about 3000 tons. The first stage was of very heavy and powerful construction, with five F1 motors powered by kerosene and liquid air, developing a thrust of 7.5 million pounds, the second stage was propelled by five T2 motors. The third stage, which contained the Apollo space craft, had one T2 motor.

3.20 Shape of the rocket

During the upward flight through the denser layers of the atmosphere the components of the rocket are subjected to intense air-pressure, and also a lot of heat is produced due to viscous friction of the air. Both these factors are taken into account while designing a rocket. The body of the rocket is made cylindrical and elongated like the shape of the cigar and its nose is sharply pointed so as to reduce the air-pressure on its individual parts to the very minimum. Its frame is made of a heat resisting material and its velocity during the first part of its flight through the denser layers of the air is kept sufficiently low.

3.21 Earthquakes

3.21.1 Seismic waves

An earthquake is caused by a portion of the rigid crust of the earth getting fractured, some distance below its surface and the consequent sudden slipping of the resulting portion. It is just a landslide on a large scale, or a re-adjustment of the earth's crust, in response to a change of pressure deep in the earth's crust, down to a distance of 100 miles or so, brought about by a variety of causes, like erosion, deposition, tidal forces, centrifugal forces, etc. An earthquake thus represents the energy released by this relive motion of portions of the earth's crust.

The place where the actual fracture occurs is called the **focus of the earthquake**. The point nearest to the focus, on the surface of the earth, is called the **epicenter**.

From the focus, originate a number of different types of waves, collectively called seismic waves, which spread out to different points on the surface of the earth and which we feel as **'earthquake tremors'** . The general pattern of these seismic waves consists of the following different types of waves: (i) Preliminary waves and (ii) Surface waves.

Preliminary waves: These consist of the primary or P waves and secondary or S waves.

(a) The primary or P waves: The first to arrive at the Observing station, these are longitudinal waves, in which the particles of the earth vibrate about their mean position, along the direction of the waves themselves.

If the earth be regarded to be a homogeneous sphere, these waves, starting from the focus, travel along the chord of huge circle of the earth, with a velocity equal to $\sqrt{j/\Delta}$, where j is what is called the 'elongational elasticity '* of the earth and Δ, its density. These waves are also variously called as 'condensational', 'irrotational' and the 'push' waves and their velocity is found to be about 5 miles per second.

(b) The Secondary or S waves: These are transverse waves, in which the particles of the earth vibrate at right angles to the direction of propagation of the waves, thus having no component along this direction. Starting from the focus, these waves also travel along a chord of a huge circle of the earth and are the next to arrive at the observing Station, which a velocity equal to $\sqrt{n/\Delta}\Delta$

where n and Δ represent the modulus of rigidity and the density of the earth respectively. The other names given to these waves are 'distortional' and 'shake' waves, their velocity being about 3 miles per second.

Surface waves: They also consist of two sets of waves, viz., Rayleigh waves and Love waves.

(i) **Rayleigh Waves.**

Discovered by Lord Rayleigh, these waves are found to remain confined to a comparatively thin layer in the close vicinity of the earth's surface. They start from the epicenter and arrive at the Observing Station along a huge circle of the earth,-the displacement of the particles and any point on the earth's surface, due to them, being in the vertical plane containing their direction of propagation. Resolving this displacement we have (i) a vertical component and (ii) a horizontal component along the direction of propagation, there being no horizontal component, at right angles to it. These waves thus persist over long distances along the surface of the earth, and are almost unique in this respect. If the earth were a homogeneous sphere, these waves also would travel with a constant velocity, but due to its heterogeneous character, each single wave, starting from the epicenter, gets split up into a number of different sets of waves, each set having a different wavelength, velocity etc.; so that, what we receive at the Observing Station is a series of oscillations instead of one single 'kick or throw' as would be the case if there were no such splitting up of the original wave, i.e., if the earth were really homogeneous in composition.

(ii) **Love waves.**

The heterogeneity of the layers of the earth is also responsible for another type of surface waves, known as Love waves, in which the displacement of the earth is horizontal, but transverse to the direction of their propagation. The velocity of these waves is less in the earth's crust than in the matter below. Immediately after an earthquake, oscillations, corresponding to these waves, can be detected at almost any place on the surface of the earth.

Unlike P and S waves, which are separately and distinctly received and recorded at the Observing Station, these waves get intermingled with Rayleigh waves to form a somewhat complicated system of waves, (not yet properly understood), called long or L waves, or the main shock, registering themselves as a long series of oscillations.

3.21.2 Seismology

The study of the seismic waves constitutes what is called the science of Seismology. Prof. John Milne predicted that 'every large earthquake might be, with proper appliances, recorded at any point on the land surface of the globe'. A curious incident confirmed his prophetic words. For, a delicate horizontal pendulum, set up for the measurement of the gravitation- action of the moon, gave recordings, which turned out to be due to an earthquake, with its origin somewhere in Japan. This started a new era of intensive researches on the subject, with Prof. Milne in the very forefront; and, in 1895, he set up his own observatory at Shide in the Isle of Wight, which became the centre of a world-wide seismic survey. His reports form a fascinating and a detailed study of the growth and development of the present-day science of Seismology.

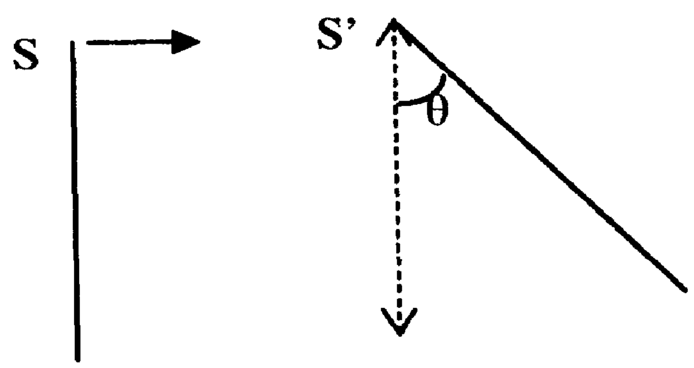
3.22 Seismographs

A seismograph (or a seismometer), is an instrument used to record the earth tremors or the seismic waves, to some dynamical function of which, (like displacement, velocity, acceleration, etc.), they respond or react. The record of the vibrations so obtained is called the **seismogram**. The instruments, responding to displacement, are of the mechanical type and we are, therefore, concerned here only with those. The following is, in brief, the theory underlying the mechanical type of seismographs.

All vibrations of the earth may ultimately be resolved into (i) vertical and (ii) horizontal components. The problem thus reduces itself to merely recording these vertical and horizontal vibrations. We shall confine our attention here only to the measurement of the horizontal displacements, accompanying these latter vibrations. There are 'two types of instruments in use for the purpose, viz., (a) the vertical pendulum and (b) the horizontal pendulum type.

(a) The Vertical Pendulum Seismographs

A vertical pendulum is just a rigid body, suspended from a stand resting firmly on the ground; so that, with the horizontal displacement of the ground and the stand with it, the point of support of the pendulum also gets displaced horizontally.



Thus, if the point of support S of the vertical pendulum is displaced horizontally to S', due to the horizontal displacement of the ground, it can be shown that a style or pen attached to its lower end, reproduces faithfully the movements of the support, with precisely the same frequency, (though on a different scale), it being assumed that the support moves with a definite frequency and amplitude.

These vertical pendulum seismographs, however, suffer from two defects, viz., (i) they have to be very heavy, as much as 22 tons or more, if a good magnification of the vibrations be desired, and (ii) their period of vibration is rather small.

(b) The Horizontal Pendulum Seismographs

We are already familiar with the horizontal pendulum, only some slight additions to its convert it into a sensitive and a reliable seismograph. With the horizontal movement of the earth, the supports of the pendulum, which are firmly fixed on to it, also share its movement, thus setting its stem or 'boom' into motion, which can then be magnified mechanically or electrically by various devices.

3.23 Modern Applications of Seismology

The development of the modern science of seismology has led to its application in four important fields, viz., (i) investigation of the nature of the interior of the earth, (ii) prospecting for oils and minerals, (iii) construction of quake-proof buildings, and (iv) forecasting of the occurrence of earthquakes.

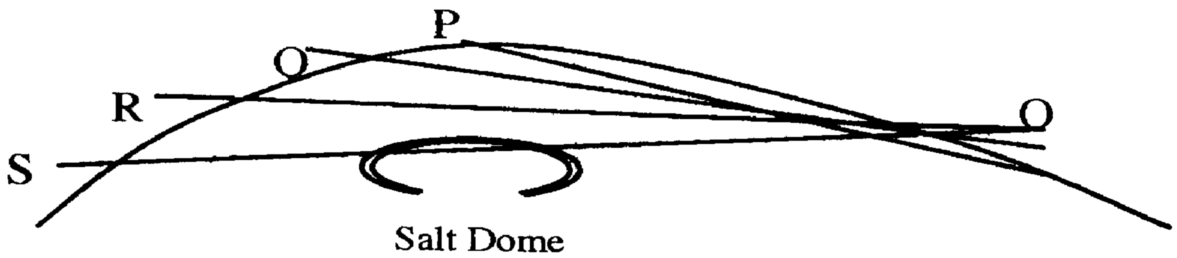
(i) Nature of the interior of the earth

It is now almost fully established that the earth consists of a dense core of a molten mass, mostly of iron, together with some nickel, of a density of about 12.0gm/cc(at the pressure existing there), surrounded by a solid outer shell or crust, about 3,000 kilometers thick, the density of which decreases from about 5.0gm/cc at its innermost layers to about 2.7gm/ccat the outermost layers, or at the surface of the earth.

The existence of the dense core is deduced from the observed refraction of the seismic waves, as they pass through the earth, andis further confirmed by the production and propagation of the secondary or shake waves (S) through the core. These waves, as we know, are transverse in nature and, as such, can only be produced and propagated in media, possessing elasticity of shape or rigidity, viz., in solids.

(ii) Prospecting for oils and minerals

Prospecting for, oil, coal and other minerals is now being increasingly done with the help of seismic waves, the process being technically known as 'seismic prospecting'.



Artificial earthquakes as set up in the ground-region to be surveyed for the purpose, by detonating an explosive, like gun-cotton or gelignite, at a point O on the earth's surface, (Figure), and the time of explosion noted. The time of arrival of the first low frequency longitudinal waves, or the primary waves, thus produced, is noted, with the help of seismographs, at different stations P, Q, R, S, etc., all lying in the same plane. The distances from O, covered along the chords OP, OQ, OR, OS etc. of the earth are carefully measured and the mean velocities of the waves calculated along these different paths or chords.

If one of the paths or chords, say, OS, happens to pass through a mineral deposit, like a salt dome, the value of the mean velocity along this particular chord will be different from that along the other chords. The experiment is then repeated along a direction, perpendicular to the first, by exploding a fresh charge of explosives. And, if this confirms the results of the first experiment, a more elaborate survey determines the positions of the top and the sides of the salt dome.

(iii) Construction of quake-proof buildings:

It has now been found possible to erect 'quake-proof buildings in California, Japan and other places, frequently visited by earthquakes, at a surprisingly low additional cost of just 15%. For, it has been shown by Prof. Suyehiro that the severest earthquakes ofJapan can do but little damage to buildings, designed to resist a horizontal force, equal to one-tenth of their total weight. The day is thus not far off when damage to buildings due to earthquakes will justbecome a memory of a dreadful past.

(iv) Forecasting of the occurrence of earthquakes:

The prediction of the occurrence of an earthquake, a good time in advance, is now fast coming into the realm of practical possibility. For, it has now been established that the region, where an earthquake occurs, exhibits, for quite a few years before, a 'tilt', or a gradual rise, very much like the rubber tube of a pneumatic tire or a football bladder swelling up before it actually bursts.

There seems to be but little doubt that much sooner than we can imagine at the moment, an earthquake forecast will become as general and universal an affair as the weather forecast is today.

But even as it is, the loss in buildings etc., resulting from the severest earthquakes seldom exceeds 5%, due to their being confined to a very small area, and, quite often, an uninhabited one. The disastrous effects of earthquakes have thus been unduly magnified; and, for all we know, they may be for our own good, designed by a benign Providence, by way of safety devices to save us from being blown up, all in a heap.

EXERCISE III

PART A

1. State Newton's law of gravitation.
2. State any kepler's III law of planetary motion.
3. Write down the expressions for mass and density of earth.
4. What is seismology?
5. Write down any two applications of seismology.
6. Define orbital velocity.
7. Define escape velocity
8. Write down the relation between orbital and escape velocity.
9. Write the principle of rocket motion.
10. Define specific impulse.
11. Write a note on multistage rocket.

PART B

1. State and explain kepler's laws of planetary motion.
2. Derive an expression for the gravitational field due to a spherical shell.
3. Explain how 'g' varies with altitude?
4. Expalin how 'g' varies with depth?
5. Describe about vertical pendulum seismograph.
6. Derive an expression for the orbital velocity.
7. Obtain an expression for the escape velocity.
8. Explain about stationary satellite.
9. Derive an expression for the velocity of rocket at any instant.
10. Explain Rocket propulsion systems.

PART C

1. Derive an expression for the gravitational potential due to a spherical shell at a point outside the shell and on its surface.
2. Define Gravitational constant. Explain Boy's method to determine the same.
3. Explain the variation of 'g' with latitude.
4. Explain about various types of waves in seismology.
5. Describe the principle and theory of rocket motion with neat diagram.
6. Explain in detail about multistage rocket.

UNIT IV: ELASTICITY

Structure

4.1 Introduction

The theory of elasticity deals with the deformations of elastic solids and has a well developed mathematical basis. When an elastic material is deformed due to an external force, it experiences internal forces that oppose the deformation and restore it to its original state if the external force is no longer applied. There are various elastic moduli, such as Young's modulus, the shear modulus, and the bulk modulus, all of which are measures of the inherent stiffness of a material as a resistance to deformation under an applied load. The various moduli apply to different kinds of deformation. For instance, Young's modulus applies to uniform extension, whereas the shear modulus applies to shearing.

The elasticity of materials is described by a stress-strain curve, which shows the relation between stress (the average restorative internal force per unit area) and strain (the relative deformation). For most metals or crystalline materials, the curve is linear for small deformations, and so the stress-strain relationship can adequately be described by Hooke's law and higher-order terms can be ignored. However, for larger stresses beyond the elastic limit, the relation is no longer linear. For even higher stresses, materials exhibit plastic behavior, that is, they deform irreversibly and do not return to their original shape after stress is no longer applied. For rubber-like materials such as elastomers, the gradient of the stress-strain curve increases with stress, meaning that rubbers progressively become more difficult to stretch, while for most metals, the gradient decreases at very high stresses, meaning that they progressively become easier to stretch. Elasticity is not exhibited only by solids; non-Newtonian fluids, such as visco-elastic fluids, will also exhibit elasticity in certain conditions. In response to a small, rapidly applied and removed strain, these fluids may deform and then return to their original shape. Under larger strains, or strains applied for longer periods of time, these fluids may start to flow like a viscous liquid.

4.2 Objective

This chapter deals with theory and application aspects of elasticity and will include :

1. Definition of stresses, strains, equilibrium and compatibility.
2. Derivation of the governing equations.
3. Solution of problems in plane stress, plane strain, torsion, bending.
4. Introduction to three-dimensional problems.

The chapter intends to provide the student with the tools and an understanding of the deformation and motion of elastic solids, the formulation of the governing equations using physical laws, and the solution of simple linear elasticity problems using various analytical techniques.

4.3 Elasticity

When an external force is applied to a body, the body gets deformed in shape or size.The property by virtue of which a deformed body tends to regain its original shape and size after the removal of deforming forces s called elasticity.

4.4 Stress and strain

Stress: Stress is defined as the restoring force per unit area.

Suppose a force F is applied normally to the area of cross – section A of a wire.

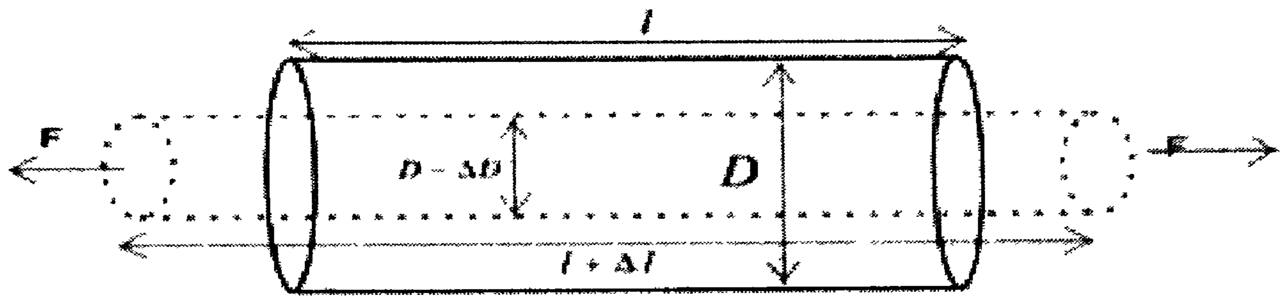
$$\text{Stress} = \frac{\text{Force}}{\text{Area}} = \frac{F}{A}.$$

Unit of stress is N m^{-2}

Strain: When a deforming force is applied, there is a change in length, shape or volume of the body. The ratio of the change in any dimension to its original value is called strain.

4.5 Poisson's ratio (γ)

When a force is applied along the length of wire, the wire elongates along the length but it contracts radially (Figure).



$$\text{Longitudinal strain} = \frac{\text{increase in length}}{\text{original length}} = \frac{\Delta\Delta L}{L}$$

$$\text{Lateral strain} = \frac{\Delta\text{decrease in diameter}}{\text{original diameter}} = \frac{\Delta D}{D}$$

The poisson's ratio (v) is defined as the ratio of lateral strain to longitudinal strain.

$$\text{Poisson's ratio } \gamma = \frac{\text{Lateral strain}}{\text{Longitudinal strain}} = \frac{\Delta D/D}{\Delta L/L}$$

$$\gamma = \frac{L\,\Delta D}{D\,\Delta L}$$

4.5 Hooke's Law

Within elastic limit, the stress is directly proportional to strain.

$$\text{Stress} \propto \text{strain}$$

$$\text{Stress} = \text{E strain}$$

$$\frac{\text{stress}}{\text{strain}} = \text{E}$$

E is constant called modulus of elasticity.

The dimensional formula of modulus of elasticity is $ML^{-1}T^{-2}$.

Its units are N m^{-2}.

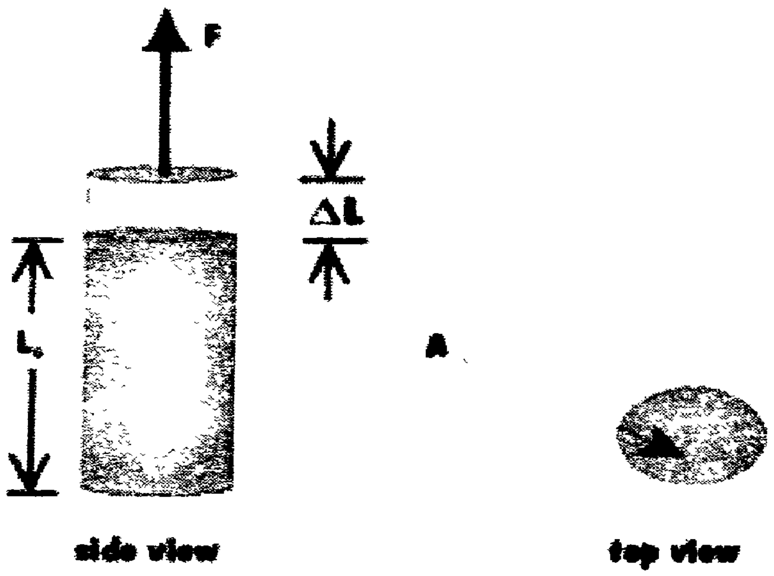
4.7 Different moduli of elasticity

Young's modulus (E)

It is defined as the ratio of longitudinal stress to longitudinal strain.

$$E = \frac{\text{Longitudinal stress}}{\text{Longitunal strain}}.$$

Consider a wire of length L and area of cross section A (figure). It undergoes an increase in length l when a stretching force F is applied.



$$\text{Longitudinal stress} = \frac{F}{A}$$

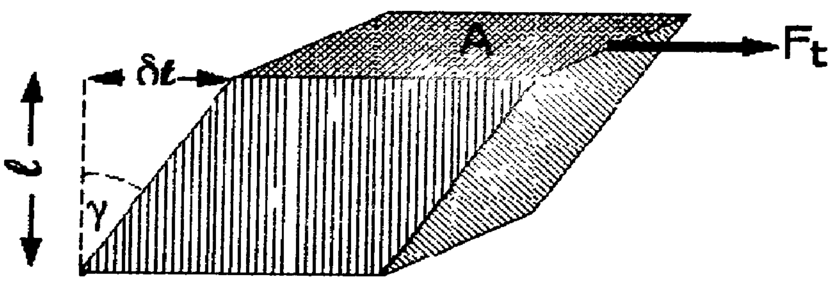
$$\text{Longitudinal strain} = \frac{\Delta L}{L}$$

$$\therefore E = \frac{F/A}{L/\Delta L} = \frac{F\Delta L}{AL}.$$

Rigidity modulus (G)

It is defined as the ratio of tangential stress to shearing strain.

$$G = \frac{\textit{Tngential stress}}{\textit{Shearing strain}}$$



Suppose the lower face of a tangential force F_t is applied at the upper face of area A (figure).

Shearing strain = γ

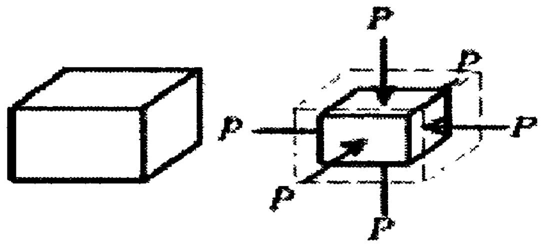
Tangential stress = $\frac{F}{A}$

Rigidity modulus (G) = $\frac{\text{Tangential stress}}{\textit{Shearing strain}} = \frac{F/A}{\gamma} = \frac{F}{A\gamma}$.

Bulk modulus (K)

It is defined as the ratio of volume stress (Bulk stress) to the volume strain.

$K = \frac{\textit{volume stress}}{\textit{volume strain}}$.



Suppose three equal stress (F/A) act on a body in mutually perpendicular directions (Figure).

There is a change of volume ΔV in its original volume V.

Bulk stress = $\frac{F}{A}$. Volume strain = $\frac{\Delta V}{V}$.

$\therefore K = \frac{\text{Bulk stress}}{\text{Volume strain}} = \frac{F/A}{\Delta v/V} = \frac{F\,V}{\Delta V\,A}$

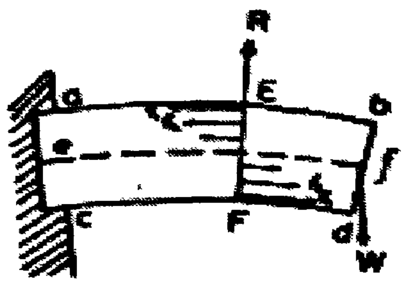
4.8 Bending of Beams

Definitions:

Beam: A beam is defined as a rod or bars of uniform cross- section (circular or rectangular) whose length is very much greater then its thickness.

4.9 Bending moment:

If a beam is fixed at one end and loaded at the other end, it bends (Figure). The load acting vertically downwards at its free end and the reaction at the support acting vertically upwards, constitute the external bending couple. A restoring couple is developed inside the beam due to its elasticity. The moment of this



elastic couple is called the internal bending moment. When the beam is in equilibrium, external bending moment = internal bending moment.

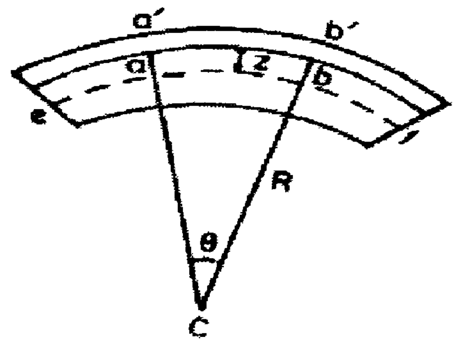
Neutral Axis:

When a beam is bent, filaments like ab in the upper part of the beam are elongated. Filaments like cd in the lower part are compressed. Therefore, there must be a filament like of in between, which is neither elongated nor compressed. Such a filament is called the natural filament. The axis of the beam lying on the neutral filament is the neutral axis. The change in length of any filament is proportional to the distance of the filament from the neutral axis.

Expression for the bending moment

Consider a portion of the beam to be bent into a circular is (figure). ef is the neutral axis. Let R be the radius of curvature of the neutral axis. θ is the angle subtended by the neutral axis at its center of curvature C.

Filaments above ef are elongated while filaments below ef are compressed. The filament ef remains unchanged in length.



Let a' b' be a filament at a distance z from the neutral axis. The length of this filament a' b' before bending is equal to that of the corresponding filament on the neutral axis ab.

Original length = ab = R θ

Its extended length = a' b' = (R+z) θ.

Increase in its length = a' b' – ab = (R+z) θ - R θ = z. θ

∴ Linear strain = $\frac{increase\ in\ length}{original\ length} = \frac{z\theta}{R\theta} = \frac{z}{R}$

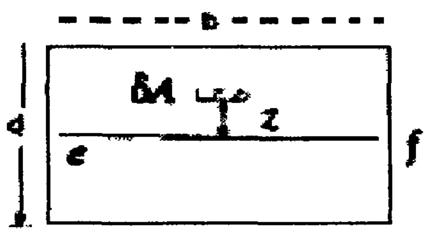
Young's modulus E = $\frac{stress}{Linear\ strain}$

Stress = E × linear strain = $E\frac{z}{R}$

Force acting on the element of area of cross – section δA (figure) is

$$F = \text{stress} \times \text{area} = \frac{Ez}{R}\delta A$$

Moment of this force about the neutral Axis = $\frac{Ez}{R}\delta A \times z = \frac{E}{R}\delta A \times z^2$



The sum of moments of forces acting on all the filaments is the internal bending moment.

Internal bending moment = $\sum\frac{E}{R}\delta A \times z^2 = \frac{E}{R}\sum \delta A \times z^2$

Σ δ A. z^2 is called geometrical moment of inertia. It is denoted by Ak^2.

Σ δ A. z^2 = Ak^2.

(A = Area of cross – section and k = radius of gyration).

∴ Internal bending moment $= \frac{EAk^2}{R}$.

Note: For a rectangular beam of breadth b, and depth (thickness) d, A = bd and

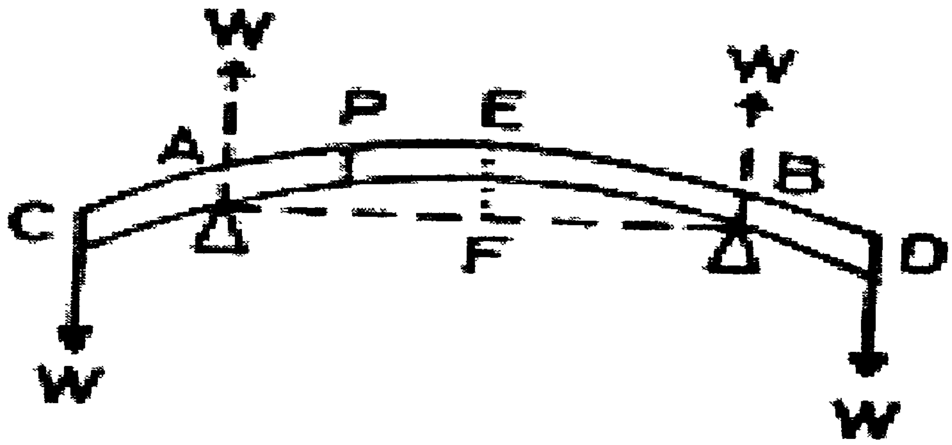
$k^2 = \frac{d^2}{12}$ ∴ Internal bending moment $= \frac{Ebd^3}{12}$.

4.10 Uniform Bending

Determination of young's modulus by Uniform Bending method

Theory

Consider a beam supported symmetrically on two knife – edges A and B (Figure). Let AB $=l$.



Equal weights W, W are suspended at its ends C and D . Let AC = BD = a.

Reactions W, W act upwards at the knife edges. The beam bends into an are of a circle of radius R. The elevation of the midpoint of the beam is EF = y.

Consider the cross – section of the beam at any point P.

The external bending moment with respect to P

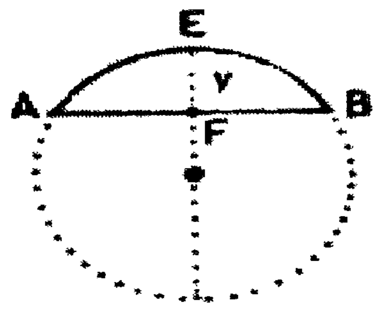
= W.CP – W. AP = W(CP-AP) = W. AC =Wa.

This must be balanced by the internal bending moment EAk^2/R.

$$\therefore \frac{EAk^2}{R} = Wa$$

$$\frac{1}{R} = \frac{Wa}{EAk^2} \quad ......(1)$$

From the property of a circle (Figure)



EF(2R – EF) = (AF)2

$y\,(2R-y) = (\frac{l}{2})^2$

$y\,2R = \frac{l^2}{4}$ [y^2 neglected]

$y = \frac{l^2}{8R}$

Substituting the value of $\frac{1}{R}$ from Eq.(1),

$y = \frac{wal^2}{8EAk^2}$ But W = mg and for a rectangular beam, $Ak^2 = \frac{bd^3}{12}$

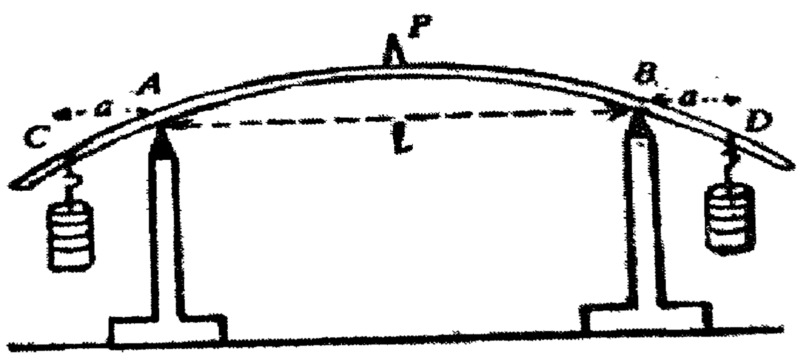
$\therefore\ y = \frac{mgal2}{8E(\frac{bd^3}{12})}$

$\therefore\ E = \frac{3mgal^2}{2bd^3\,y}$.

Experiment: Pin and microscope method

The given beam is supported symmetrically on two knife edges A and B as shown in the figure. The distance between the knife edges is *l*. two weight hangers are suspended from C and D. $AC = AD = a$

A pin is placed vertically at the center of the beam. The elevation produced at the mid-point of the beam is measured using a microscope.



The load on each hanger is increased in equal steps of *m* kg. The corresponding microscope readings are noted. Similarly the readings are noted while unloading. The results are tabulated as follows.

Load in kg	Reading of the microscope			y for M kg
	Load increasing	Load decreasing	Mean	

The mean elevation y of the centre for M kg is found. The breadth b and the thickness d of the beam are measured using vernier calipers and screw gauge respectively. The Young's modulus of the material of the beam is calculated using the formula

$$E = \frac{3Mgal^2}{2bd^3y}$$

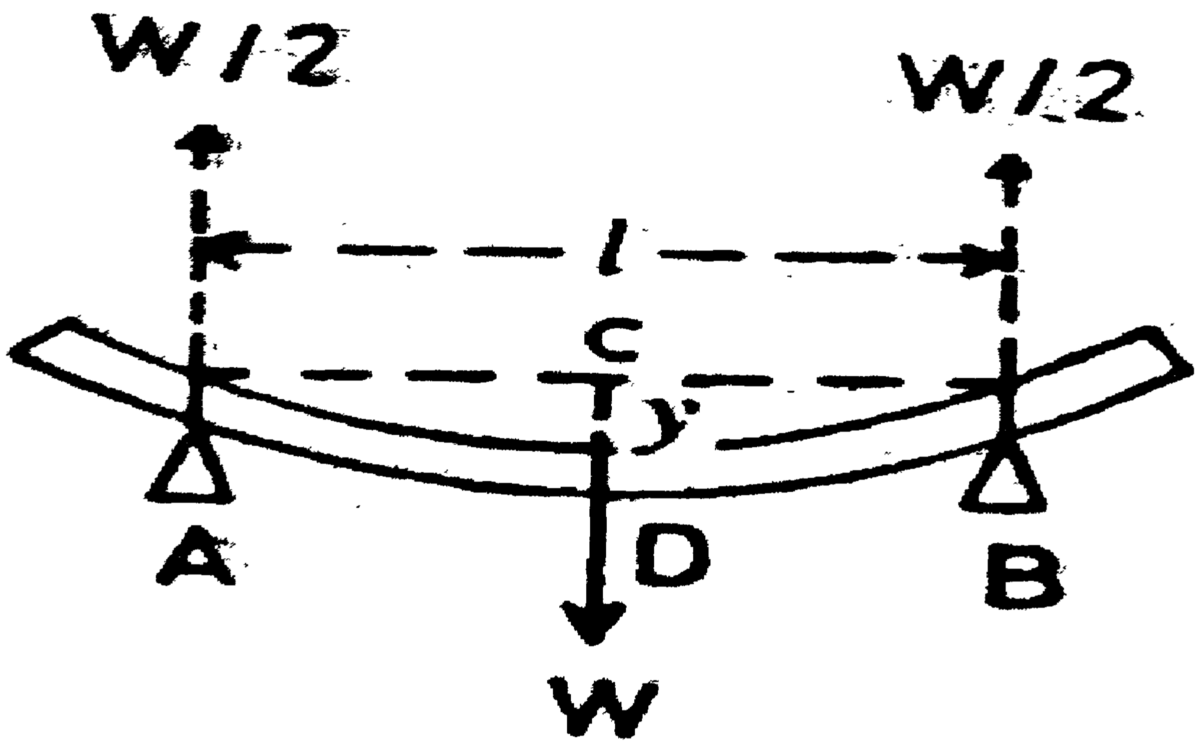
4.11 Non-Uniform Bending

Determination of young's modulus by Non-uniform Bending

Theory

AB represents a beam of length l, supported on two knife – edges at A and B (figure). A load W is suspended at the center C. The reaction at each knife – edge is W/2 acting vertically upwards. The beam bends. The depression is maximum at the center. The bending is non-uniform.

Let CD = y.



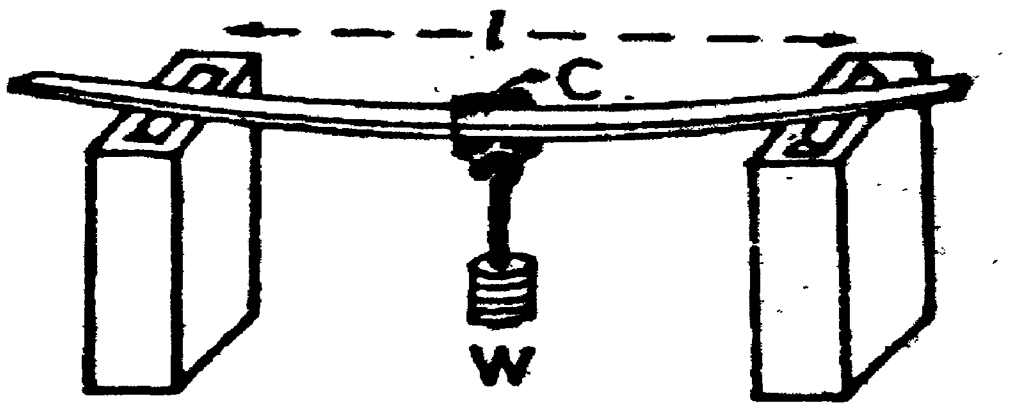
The portion DA of the beam may be considered as a cantilever of length l/2, fixed at D and bending upwards under a load W/2.

The depression of D below A = $y = \frac{(W/2)(l/2)^3}{3EAk2} = \frac{Wl^3}{48\ EAk^2}$

W = Mg. For a rectangular beam, $Ak^2 = \frac{bd^3}{12}$. Hence $y = \frac{mgl^3}{48\ E\ (\frac{bd^3}{12})}$

$$E = \frac{mgl^3}{4\ b\ d^3\ y}$$

Experiment



The given beam is symmetrically supported on two knife– edge (fig). a weight – hanger is suspended by means of a loop of thread from the center C. A pin is fixed vertically at C by some wax .A travelling microscope is focused on the tip of the pin such that the horizontal cross wire coincides with the tip of the pin. The reading in the vertical traverse scale of microscope is noted. Weights are added in equal steps of m kg. The corresponding readings are noted. Similarly, readings are noted while unloading. The results are tabulated as follows.

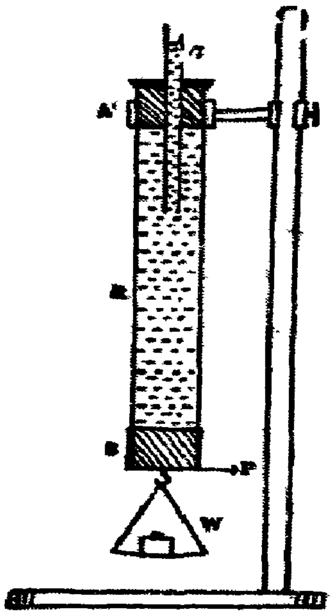
Load in kg	Reading of the microscope			y for M kg
	Load increasing	Load decreasing	Mean	

The mean depression *y* is found for a load of *M* kg. The length of the beam (*l*) between the knives- edges is measured. The breadth *b* and the thickness *d* of the beam are measured with a vernier calipers and screw gauge respectively.The Young's modulus of the material of the beam is calculated using the formula

$$E = \frac{mgl^3}{4\ b\ d^3\ y}$$

4.12 Determination of Poisson's ratio (for rubber)

To determine the value of α for rubber, we take about a metre-long tube AB of it, (Figure), such, for example, as the tube of an ordinary cycle tyre , and suspended it vertically, as shown, with its two ends properly stoppered with rubber bungs and seccotine (a liquid glue). A glass tube C, open at both ends, about half in the stopper at the upper end A, so that a major part of it projects out.



The rubber tube is completely filled with air-free water until the water rises up in the glass tube to height of about 30cm from A. A suitable weight W is now suspended from the lower end B of the tube. This naturally increases the length as well as the internal volume of the tube, resulting in a fall of the level of the meniscus in C. Both, the increase in length (dL) and the fall (dh) in the level of the meniscus in C, are measured with the help of a travelling microscope, by noting the new positions of point P and the level of the meniscus C, though, for ordinary purposes, the former may be read conveniently on a vertical metre scale, with the help of a pointer, attached to the suspension of W.

Let the original length, diameter and volume of the rubber tube be L,D and V respectively. Then, its area of cross-section

$$A = \pi\left(\frac{D}{2}\right)^2 = \frac{\pi D^2}{4} \qquad \text{........(1)}$$

Differentiating which, we have

$$dA = \frac{\square D}{2}\, dD \text{ , Whence, } dA = \frac{2AdD}{D} \text{........(2}$$

Now, if corresponding to a small increase dV in the volume of the rubber tube, the increase in its length be dL, and the decrease in its area of cross-section be dA, we have

$$V+dV = (A-dA)(L+dL)$$

$$= AL+A.dL-dA.L-dA.dL.$$

$$V + dV = V + A.dL = dA.L.$$

Where AL = V, the original volume of the tube and dA. dL is a small quantity, compared with the other terms.

So that, $dV = A.dL - dA.L = A.dL - \frac{2AL}{D}dD.$

Or, dividing both sides by dL, we have

$$\frac{dV}{dL} = A - \frac{2AL}{D}\frac{dD}{dL} = A - \frac{dv}{dL},$$

Whence, $$\frac{dD}{dL} = (A - \frac{dv}{dL}) / \frac{2AL}{D} = \frac{AD}{2AL} - \frac{dv}{dL}.\frac{D}{2AL}.$$

$$= \frac{D}{2L} - \frac{dv}{dL}\frac{D}{2AL}$$

Or, $\frac{dD}{dL} = \frac{D}{2L}(1 - \frac{1}{A}\frac{dv}{dL})$

Now, Poisson's ratio, $\gamma = \frac{\text{lateral strain}}{\text{longitudinal strain}} = \frac{dD/D}{dL/L} = \frac{dD}{dL}\frac{L}{D}$

Or, $$\gamma = \frac{L}{D}\frac{dD}{dL}$$

Or, substituting the value of dD/dL from relation (iii) above, we have

$$\gamma = \frac{L}{D}.\frac{D}{2L}(1 - \frac{1}{A}\frac{dv}{dL}) = \frac{1}{2}(1 - \frac{1}{A}\frac{dv}{dL}) \quad .....(I)$$

If r be the internal radius of cross-section of the tube, clearly,

$A = \pi r^2$ So that,

$$\gamma = \frac{1}{2}(1 - \frac{1}{\pi r^2}\frac{dv}{dL}).$$

Similarly, if a be the internal radius of the capillary tube, we have $dV = a^2.dh$, because , as we know, a fall in the level of the meniscus in C corresponds to an increase in volume of the rubber tube AB. We, therefore have

$$\gamma = \frac{1}{2}(1 - \frac{1}{\pi r2}\frac{\pi a2.dh}{dL}) = \frac{1}{2}(1\frac{a2}{r2}\frac{.dh}{dL}) \quad ......(II)$$

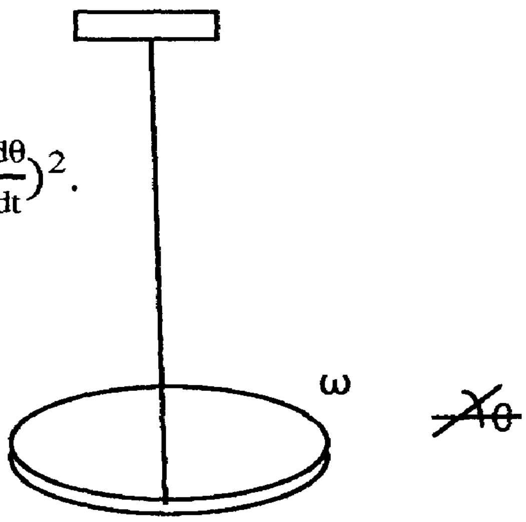
The values of a and r are determined carefully at several points by means of a travelling microscope and a vernier caliper respectively and their mean values taken. And, the average value of dh/dL is obtained from the slope of the straight-line graph by plotting a number of corresponding values of dh against those of dL, (Figure) on suspending different loads from the tube. The value of for the given specimen of rubber can then be easily calculated from relation (II).

For ordinary purposes, we may use a capillary tube graduated in cubic centimeters and read dV directly on it. Then, substituting the values of A (i.e., r2) and of dV/dL in relation I, α may be evaluated. Here too, however, it is advisable to plot dV against dL for different loads suspended from the tube and to use the average value of dV/dL given by the slope of the straight-line graph, thus obtained.

4.13 Torsional oscillations of body

Suppose a wire is clamped vertically at one end. The other end carries a disc of moment of inertia I about the wire as the axis (figure). The wire is twisted through a small angle and released. The disc executes Torsional oscillations. The arrangement is called a Torsion Pendulum.

Let us consider the energy of the vibrating system when the angle of twist is θ. Let ω be the angular velocity of the body.



The potential energy of the wire due to the twist $= \frac{1}{2}c\theta^2$

The kinetic energy of the body due to its rotation $= \frac{1}{2}I\omega^2 = \frac{1}{2}I(\frac{d\theta}{dt})^2$.

The total energy of the system $= \frac{1}{2}I(\frac{d\theta}{dt})^2 + \frac{1}{2}c\theta^2$ = constant.

Differentiating this with respect to t,

$$\frac{1}{2}I\,2\frac{d\theta}{dt}\frac{d^2\theta}{dt} + \frac{1}{2}c2\theta\frac{d\theta}{dt} = 0$$

Or $I\,\frac{d^2\theta}{dt} + c\theta = 0$ or $\frac{d^2\theta}{dt} + \frac{c}{I}\theta = 0$

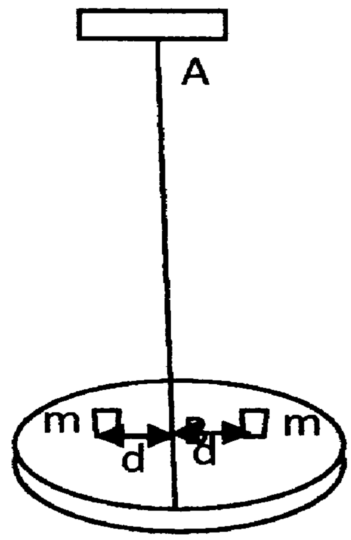
The body has simple harmonic motion. Its period is given by

$$T = 2\,\pi\sqrt{\frac{I}{c}}$$

4.14 Torsional pendulum

Rigidity modulus by Torsion pendulum

The wire AB of length L and radius a is fixed at the end A (figure). The lower end B is clamped to the centre of a circular disc. Two equal masses (each equal to m) are placed along a diameter of the disc at equal distance d_1 on either side of the center of the disc. The disc is rotated through an angle and is then released. The system executes torsional oscillations about the axis of the wire. The period of oscillations T_1 is determined.



Then, $T_1 = 2\pi\sqrt{\frac{I_1}{c}}$

Or $T_1^2 = \frac{4\pi^2}{c} I_1$.

Here, I_1 = Moment of inertia of the whole system about the axis of the wire, c = torque per unit twist.

Let I_0 = M.I. of the disc alone about the axis of the wire.

i = M.I. of each mass about a parallel axis through its center of gravity.

Then by the parallel axes theorem

$$I_1 = I_0 + 2i + 2md_1^2$$

$$\therefore T_1^2 = \frac{4\pi^2}{c}[I_0 + 2i + 2md_1^2] \quad \text{..........(1)}$$

The two masses are now kept at equal distance d_2 from the center of the disc. The corresponding period T_2 is determined. Then,

$$T_2^2 = \frac{4\pi^2}{c}[I_0 + 2i + 2md_2^2] \text{ ....(2)}$$

$$\therefore T_2^2 - T_1^2 = \frac{4\pi^2}{c} 2m[d_2^2 - d_1^2] \quad \text{..........(3)}$$

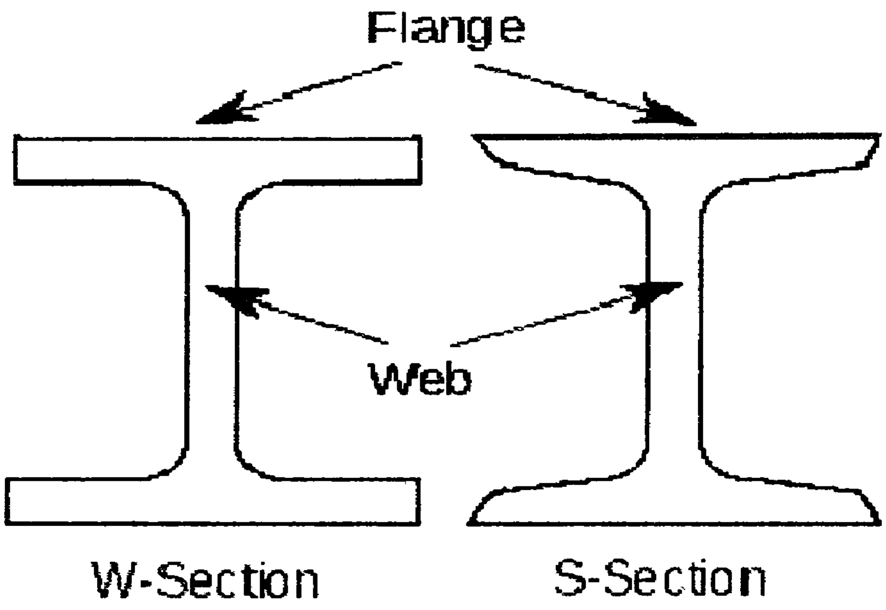
But $c = \pi Ga^4/2L$. Hence $T_2^2 - T_1^2 = \frac{4\pi^2 2L}{\pi Ga^4} 2m[d_2^2 - d_1^2]$

Or $$G = \frac{16\pi Lm(d_2^2 - d_1^2)}{(T_2^2 - T_1^2)a^4} \quad \text{.......... (4)}$$

Using this relation, rigidity modulus G is determined.

4.15 I – Section Girders

Girders standing on pillars at their ends support loads. As a result, the girder suffers bending. Beam theory shows that the I-shaped section is a very efficient form for carrying both bending and shearing loads in the plane of the web. On the other hand, the cross-section has a reduced capacity in the transverse direction, and is also inefficient in carrying torsion, for which hollow structural sections are often preferred.



When a girder is loaded, the middle portion gets depressed. The neutral surface lying in the middle of the girder experiences no strain. The filaments in the lower part suffer extension. The filaments in the upper part suffer compression. The compression or extension is proportional to the distance from the neutral surface. Hence the stresses produced in the beam are maximum at the upper and the lower surface of the beam. Consequently, the girders must be strong at the upper and the lower surfaces. This is the reason the girders used in buildings are made of I – section. Thus a large amount of material is saved.

EXERCISE IV

PART A

1. Define stress and strain.
2. State Hooke's law.
3. What is a beam?
4. Write down the formula for bending moment of a beam and explain the terms.
5. Define Poisson's ratio.
6. What are the advantages of I shaped beam?

PART B

1. Explain about different modulii of elasticity.
2. Derive an expression for bending moment.
3. Explain about bending of beams.
4. Explain the theory of uniform bending.
5. Explain the theory of Non-uniform bending.
6. Describe about the torsional oscillations of a body.

PART C

1. Explain the experiment to determine the young's modulus of the material by uniform bending method with theory.
2. Describe the experiment to determine the young's modulus of the material by non-uniform bending method with theory.
3. Explain an experiment to determine poisson's ratio of rubber with theory.
4. Describe with theory the determination of rigidity modulus of the wire using torsional pendulum.
5. Explain in detail about I- section girders.

UNIT V: FLUID MOTION, VISCOSITY AND SURFACE TENSION

STRUCTURE

5.1 Introduction

In the case of a liquid at rest, the hydrostatic pressure at any point inside the liquid in given by $P = h \rho g$, where h is the depth of the point below the liquid surface, ρ is the density of the liquid and g is the acceleration due to gravity. However, when the liquid is in a state of motion, to calculate the pressure at a point, certain other factors such as velocity of flow, potential energy etc, have to be taken in to account, besides depth and density. A fluid in motion possesses various forms of energy e. g. kinetic energy, potential energy and gravitational energy. The viscous property of the fluid and its incompressibility has to be taken into account in the case of ideal liquids, it is assumed that the viscous forces are completely absent and the liquid is highly incompressible.

5.3 Fluid

Most matter can conveniently be described as being in one of the three phases - solid, liquid or gas. Solids and liquids (also called condensed matter) have a certain properties in common, for example, they are relatively incompressible and their densities relatively constant as we very the temperature. Gases on the other hand are easily compressible and their density changes substantially with temperature.

Together, liquids and gases are classified as fluids. These materials will easily flow under the action of shearing force. We commonly observe this when a fluid flows to conform to the shape of its container.

5.3.1 Flow of a fluid

The study of fluids in motion is known as fluid dynamics. The motion of liquids and gases is very complex. Fluid flow has the following characteristics.

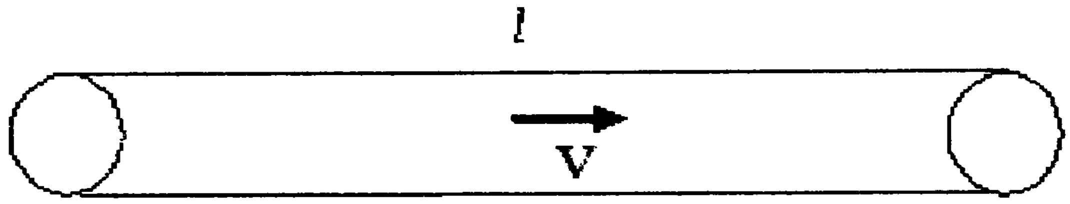
i) It can be steady or non steady ii) It can be compressible or incompressible. iii) It can be viscous or non viscous and iv) It can be rotational or irrotational.

Therefore, to study the motion of a fluid a simple theory can be developed on the following assumption.

i) Fluid friction is negligible ii) the fluid is incompressible and iii) The flow is streamlined.

A fluid which obeys these conditions is known as an ideal fluid. This is a poor idealization and does not occur in practice. However, the conclusions reached on these assumptions are applicable for fluids at low velocities.

5.3.2 Rate of flow



When a liquid flows through a pipe, the rate of flow is the volume of liquid flowing across any section in unit time.

Let an ideal liquid (in compressible and frictionless) flow with uniform velocity v through a tube of cross section ‘a’. Let l be the distance covered in time t. The volume of liquid flowing is time t is = = avt

∴ Volume of liquid flowing per second = $\frac{}{t}$ =.

∴ Rate of flow = .

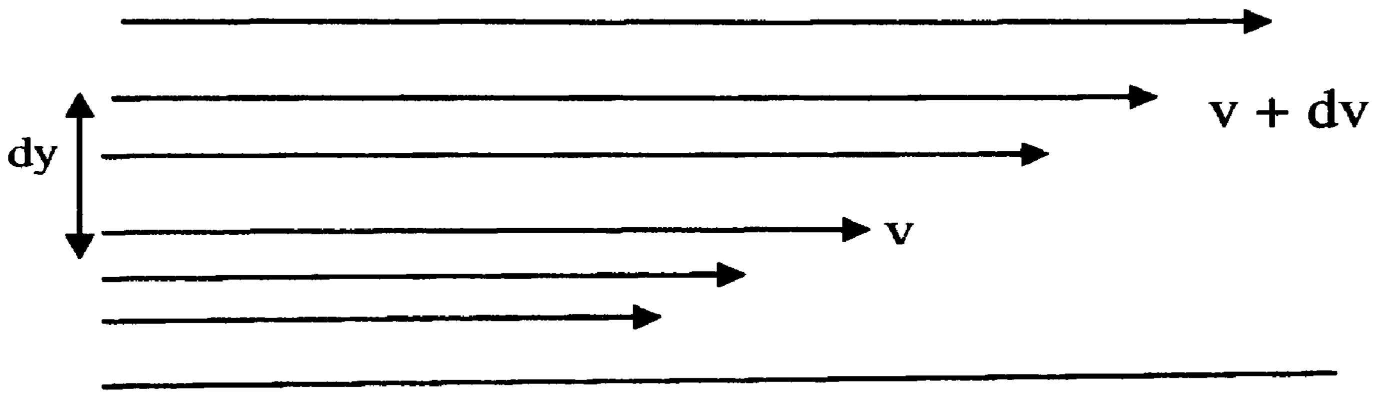
5.4 Viscosity

When water, kerosene or alcohol is poured from a bottle they are found to easily flow out. But when we try to pour honey or castor oil, these liquids do not easily flow out of the bottle. A force of liquid friction exists between the different layers of the liquid. This property of the liquid due to which the relative motion between the different layers of the liquid is prevented is known as viscosity. Honey and castor oil are called highly viscous liquids.

5.4.1 Coefficient of viscosity

Let us consider a liquid flowing along a horizontal surface. The layer of liquid which is in contact with the surface is at rest. The next layer moves with a small velocity. The topmost layer has maximum velocity (figure).

The intermediate layers possess velocities which gradually increase with height. The variation of velocity with distance (—) is known as the velocity gradient



If we consider two successive layers of the liquid, the upper layer will try to accelerate the motion of the lower layer. At the same time the lower layer will tend to retard the motion of the upper layer. There is a force of liquid friction which prevents relative motion between the two layers. This force is called viscous force. This viscous force is (i) directly proportional to the area of contact A between the layers and (ii) directly proportional to the velocity gradient $\left(\frac{dv}{dy}\right)$

$$\therefore F \propto A\frac{dv}{dy} \qquad \text{or} \qquad F = \eta A\frac{dv}{dy}$$

Where η is known as the coefficient of viscosity of the liquid

When A = 1 and $\frac{dv}{dy} = 1$, $F = \eta$. Therefore the coefficient of viscosity of a liquid is defined as the tangential force acting on unit area of the liquid layer maintaining in it a unit velocity gradient normal to the liquid layer.

Viscosity is measured in Nsm^{-2}. Its dimensions are $ML^{-1}T^{-1}$.

5.5 Critical Velocity

Critical velocity of a liquid is defined as the velocity below which the motion of the liquid is streamlined and above which the motion of the liquid becomes turbulent.

Deduction of the Expression for the critical velocity by the method of dimensions

The critical velocity of a liquid may depend upon (i) the coefficient of viscosity of the liquid (η) (ii) the density of the liquid (ρ) and (iii) the radius 'r' of the tube through which the liquid is flowing. We may write

$$v_c = k\,\eta^a \rho^b r^c$$

where k is a dimensionless number called Reynolds' number. Writing the dimensions of these quantities we have.

$$[LT^{-1}] = [ML^{-1}T^{-1}]^a[ML^{-s}]^b[L]^c$$

$$[LT^{-1}] = [M^{a+b} L^{-a-3b+c} T^{-a}]$$

$\therefore$ a + b =0; -a-3b+c = 1 and –a=-1.

From these equations we have a=1, b= -1 and c = -1

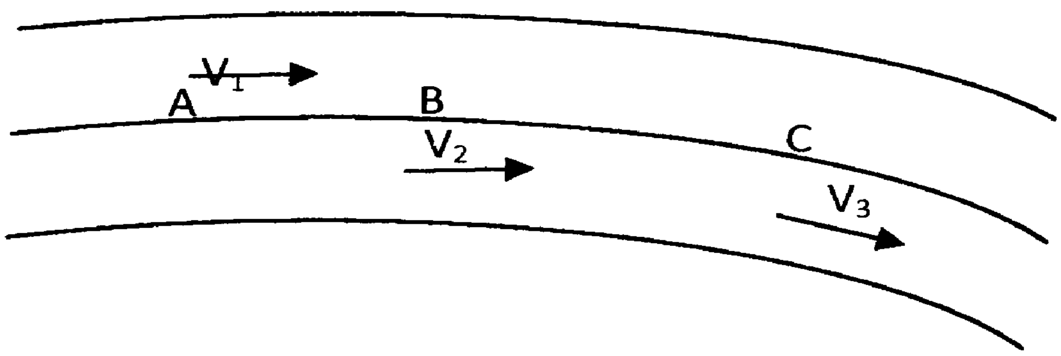
$\therefore V_c = \frac{\quad}{\rho r}$ (For narrow tubes k = 1000).

Thus the critical velocity of a liquid is directly proportional to its viscosity (η) and inversely proportional to its density and the radius of the tube. This expression indicates that narrow tubes, low density and high viscosity help in producing steady or orderly motion and wide tubes, high density and low viscosity tend to produce turbulent motion. The radio η/□ is called kinematic viscosity.

5.6 Streamlined flow and turbulent flow:

Consider a liquid flowing in a pipe. Let the velocity of flow be V_1at A, V_2at B and V_3 at C (figure). If as time passes the velocities at A, B and C are constant in magnitude and direction, the flow is said to be steady.

In a steady flow each particle follows exactly the same path and has exactly the same velocity as it predecessor. In such a case the liquid is said to have an orderly or a streamlined flow. The line ABC is called a stream line which is the path followed by an orderly procession of particles. The tangent to the stream-line at any point gives the velocity of the liquid at the point.



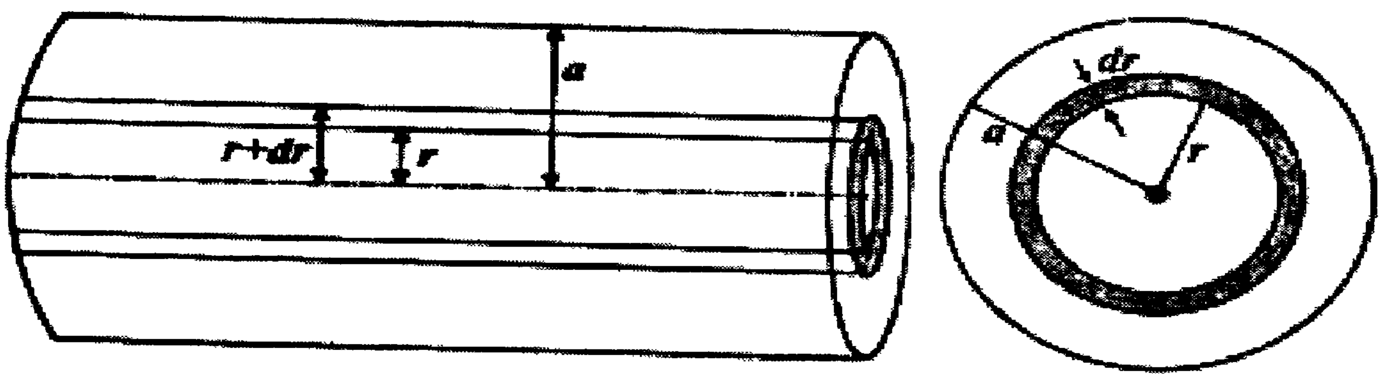
The flow is steady or stream-lined only as long as the velocity of the liquid does not exceed a limiting value. Called the flow of the liquid is excessive, the motion of the liquid takes place with a velocity greater than the critical velocity and the motion becomes unsteady or turbulent. This causes eddies and whirlpools in the motion of the liquid. This turbulent motion is also known as vortex motion.

The distinction between stream-line flow and turbulent flow can be demonstrated by injecting a jet of ink axially in a wider tube in which water is made to flow axially, when the velocity of the liquid is small, the ink will move in a straight line. As the speed of flow is increased beyond the critical velocity, the ink will spread out showing that the motion has become turbulent.

5.7 Poiseuille's equation for flow of liquid through a tube

Poiseuille's formula

An expression for the volume of liquid flowing through a horizontal capillary tube in one second can be derived considering the flow of liquid to be stream lined. This formula can be used for determining the coefficient of viscosity of liquids.



Let us consider or horizontal capillary tube of length l and radius a through which a liquid flows (figure). Let η be the coefficient of viscosity of the liquid and P the difference of pressure between the ends of the capillary tube.

The velocity of the liquid is maximum along the axis of the tube and is zero at the walls. Let $\frac{dv}{dr}$ give the velocity gradient.

Let us consider a cylindrical shell of the liquid having an inner radius r and outer radius (r + dr). The shell is co-axial with the tube.

The surface area of the shell A = $2\pi rl$

The viscous force acting on this layer is given by

$$F = -\eta\, A \frac{dv}{dr} = -\eta\, 2\pi r l \frac{dv}{dr}$$

The negative sign shows that viscous force is a backward dragging tangential force. The force that is responsible for driving the liquid through the tube is provided by the difference in pressure P between the two ends.

Therefore F = Pressure × Area

$$= P \times \pi r^2$$

When the motion is steady, the backward dragging force and the driving force are equal. Therefore,

$$-\eta 2\pi r l \frac{dv}{dr} = P\pi r^2$$

$$dv = \frac{-P\, r\, dr}{2\eta l}$$

Integrating, $v = \frac{-P}{2\eta l}\frac{r^2}{2} + \text{constant}$

The layer of the liquid in contact with the inner wall of the capillary tube is at rest.

∴ When r = a, v = 0

i.e., $\frac{-P}{2\eta l}\frac{a^2}{2} + \text{constant} = 0$

or constant $= \frac{Pa^2}{4\eta l}$

Therefore velocity $v = -\frac{Pr^2}{4\eta l} + \frac{Pa^2}{4\eta l} = \frac{P}{4\eta l}(a^2 - r^2)$

This gives the average velocity of the liquid flowing through the tube

Volume of liquid flowing per second through the shell of radius r and thickness dr

= (Area of cross section of the shell) × (velocity)

$$dV = [2\pi\, r\, dr]\frac{P(a^2 - r^2)}{4\eta l} \qquad = \frac{\pi P(a^2 r - r^3)}{2\eta l}dr$$

The total volume of the liquid flowing out of the tube in one second is obtained by integrating the above expression between the limits r = 0 and r = a.

$$V = \int_0^a \frac{\pi P(a^2 r - r^3)}{2\eta l}dr$$

$$= \frac{\pi P}{2\eta l}\left[a^2\frac{r^2}{2} - \frac{r^4}{4}\right]_0^a = \frac{\pi P}{2\eta l}\left[\frac{a^4}{4}\right]$$

$$V = \frac{\pi P a^4}{8\eta l}$$

Flow of liquids

The study fluids in motion are known as fluid dynamics. The motion of liquids and gases is very complex and therefore simple theory can be developed only on the following assumption.

I) Fluid friction is negligible
II) The fluid is incompressible
III) The flow is streamlined

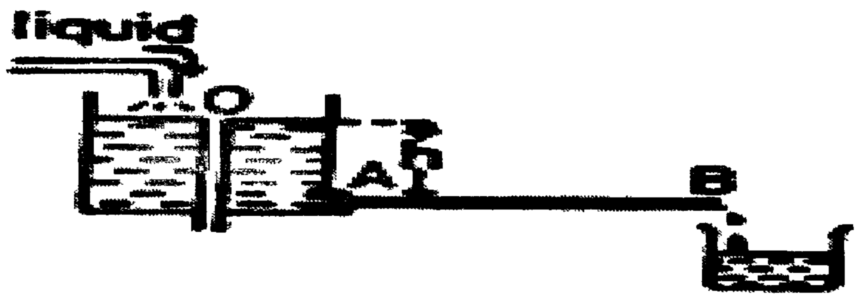
A fluid which obeys these conditions is known as an ideal fluid. This is a poor idealization and does not occur in practice. However the conclusions reached on these assumptions are applicable for flow at low velocities.

Streamlined flow

Let us consider the macroscopic motion of a fluid. (i.e., we neglect the random thermal motions of the molecules). Let us examine the average motion of small volume elements of the fluid. Suppose that a volume element passes through the point Q with a velocity v (figure). If another volume element also passes through the point Q with the same velocity then the velocity at the position Q of space does not change in time. Such a flow is termed as steady flow. If we trace out the path followed by one volume element we will find a curved trajectory. This line is called a streamline. Each volume element follows a streamline and a velocity vector is associated with each point on the streamline. The streamline may converge or diverge; but they never cross one another. A bundle of streamlines is often referred to as a tube of flow.

5.9 Experimental determination of η

<u>Poiseuille's method:</u>



The liquid is taken in the constant level tank upto a height 'h' (figure). A capillary tube, AB is fixed to the bottom of the tank. A weighted beaker is placed below the free end B of the capillary tube and the weight 'w' of the liquid collected in it in time 't' is founded out. Volume of liquid flowing per second = v = w/ρt, where ρ is the density of the liquid. The length 'l' of the capillary tube is measured by a metre scale. The radius 'a' of the capillary tube is determined very accurately using the travelling microscope. Then, from the relation,

$$\eta = \frac{\pi P a^4}{8Vl} \qquad \text{(where } p = h\rho g\text{)},$$

the value of η for the liquid can be easily calculated.

5.10 Motion in a viscous medium

Let us consider an infinite column of a highly viscous liquid like castor oil contained in a tall jar. If a steel ball is dropped into the liquid, it begins to move down with acceleration under gravitational pull. But its motion in the liquid is opposed by viscous forces in the liquid. These viscous forces increase as the velocity of the ball increases. Finally a velocity will be attained when the apparent weight of the ball becomes equal to the retarding viscous forces acting on it. At this stage the resultant force on the ball is zero. Therefore the ball continues to move down with the same velocity thereafter. This uniform velocity is called the terminal velocity.

Stokes' formula:

The viscous force F experienced by a falling sphere must depend on

(i) the terminal velocity 'v' of the ball
(ii) the radius ('r') of the ball and
(iii) the coefficient of viscosity (η) of the liquid. We can then write

$$F = kv^a r^b \eta^c \qquad \text{where k is a dimensionless constant.}$$

The dimensions of these quantities are $F = MLT^{-2}$; $v = LT^{-1}$; $r = L$; $\eta = ML^{-1}T^{-1}$; (k is a number. It has no dimensions.)

Therefore, $MLT^{-2} = (LT^{-1})^a L^b (ML^{-1}T^{-1})^c$

$$MLT^{-2} = M^c L^{a+b-c} T^{-a-c}$$

Equating the powers of M, L and T on either side, we get

$$c = 1\,;\, a + b - c = 1 \text{ and } -a - c = -2$$

Solving, a=1,b=1 and c=1. Therefore, $F = k\ v\ r\eta$.

Stokes experimentally found the value of 'k' = 6π..

Hence $\quad F = 6\pi v r \boldsymbol{\eta}$. $\quad$ This is Stokes' law.

Expression for terminal velocity:

Let **ρ** be the density of the ball and **ρ'** the density of the liquid.

Then, the weight of the ball $= \frac{4}{3}\pi r^3 \rho g$

The weight of the displaced liquid or the up thrust on the ball $= \frac{4}{3}\pi r^3 \rho' g$

The apparent weight of the ball $= \frac{4}{3}\pi r^3 \rho g - \frac{4}{3}\pi r^3 \rho' g$

$$= \frac{4}{3}\pi r^3 (\rho - \rho')g$$

When the ball attains its terminal velocity 'v',

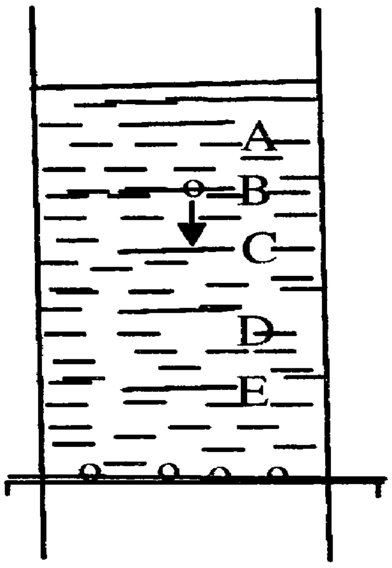
The apparent weight of the ball = viscous force F,

$$\therefore 6\pi\eta r v = \frac{4}{3}\pi r^3 (\rho - \rho')g$$

Or, terminal velocity $\quad v = \frac{2}{9}\frac{r^2}{\eta}(\rho - \rho')g$

5.11 Stokes method for the coefficient of viscosity of a viscous liquid

Stokes' method is suitable for highly viscous liquids like castor oil and glycerin. The experimental liquid is taken in a tall and wide jar (figure). Four or five marks A, B, C, D.....are drawn on the outside of the jar at intervals of 5cm. A steel ball is gently dropped centrally into the jar. The times taken by the ball to move through the distances AB, BC, CD......are noted. When the times for two consecutive transits are equal, the ball has reached terminal velocity. Now another ball is gently dropped into the jar. When the ball just reaches a mark below the terminal stage, the time (t) taken by the ball to move through a definite distance ('x') is noted. ∴ Terminal velocity = v = x/t. The experiment is repeated or various distance and the mean value of '**η**' is found.



The radius of the ball is measured accurately with a screw gauge. The density of the ball **ρ**' are found by the principle of Archimedes. □ is calculated using the formula $\eta = \frac{2}{9}\frac{r2}{v}(\rho\text{-}\rho')\,g$.

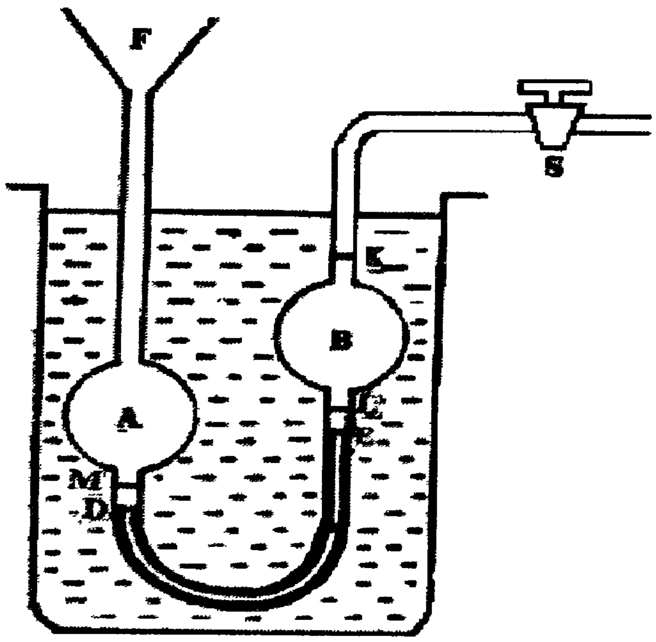
Application:

Stokes' formula is used for the determination of the electronic charge by Millikan's method.

5.12 Ostwald viscometer

This instrument is used to compare the viscosities of two liquids. It is also used to study the variation of viscosity of a liquid with temperature.

The apparatus consists of two glass bulbs A and B joined by a capillary tube DE bent into a U form (figure). The bulb A is connected to a funnel F. The bulb B is connected to an exhaust pump through a stop-cook S. K,L and are fixed marks as shown in figure. The whole apparatus is placed inside a constant temperature bath.



The liquid is then introduced into the apparatus through the funnel and its volume is adjusted so that the liquid occupies the portion between the marks K and M when the stop-cock is closed. The stop-cock is now opened and with the help of the exhaust pump the liquid is sucked up above the mark K. The stop-cock is closed and the exhaust pump is removed. The stop-cock is again opened. The liquid is allowed to flow through the capillary tube. The time (t_1) taken by the liquid to fall from the mark K to the mark L is noted. The experiment is then repeated with the second liquid and the time (t_2) taken by it to fall from K to L is noted.

Theory:

Let η_1 and η_2 be the coefficients of viscosity and ρ_1 and ρ_2 be the densities of the two liquids respectively. Let the volume of liquid between K and L be V.

Then, the rate of flow of the first liquid $v_1 = \frac{v}{t_1}$ .... (1) and

The rate of flow of the second liquid $v_2 = \frac{v}{t_2}$ .... (2)

Now, $\eta_1 = \frac{\pi P_1 a^4}{8 v_1 l}$ and $\eta_2 = \frac{\pi P_2 a^4}{8 v_2 l}$

$$\frac{\eta_1}{\eta_2} = \frac{v_2}{v_1} \times \frac{P_1}{P_2} \qquad .... (3)$$

But the pressure P is proportional to the density of the liquid used [P = h□g].

Hence, $\frac{P_1}{P_2} = \frac{\rho_1}{\rho_2}$ .... (4)

Also, dividing (2) by (1), $\frac{v_1}{v_2} = \frac{t_1}{t_2}$ .... (5)

Hence, $\frac{\eta_1}{\eta_2} = \frac{t_1 \rho_1}{t_2 \rho_2}$ .... (6)

From equation (6), $\frac{\eta_1}{\eta_2}$ can be calculated.

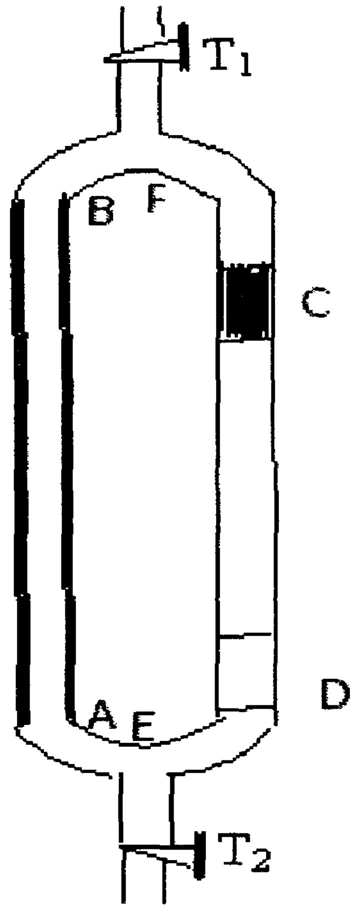
5.13 Determination of viscosity of gases

Viscosity of gases

The above methods for determining the coefficient of viscosity of liquids can also be used in the case of gases with certain modifications. Liquids are practically incompressible and hence the density of the liquid almost remains constant irrespective of the changes in pressure. But in the case of gases the density of gas varies with pressure. For liquids, the volume (or the mass) flowing per second across any cross-section is constant. In the case of gases, the mass(not volume) of the gas flowing per second at any cross-section is constant.

Rankine's method for determination of η of a gas

Rankin's apparatus consists of a closed vessel ABCD (figure). Between A and B there is a capillary tube of length l and radius a. In the opposite branch, there is a mercury pellet of mass m. There are two fixed marks C and D such that the volume v of the upper portion BC is equal to the volume of the lower portion AD. Let the volume of the gas in the whole tube ABCD be V. T_1 and T_2 are taps for



filling the vessel with the experimental gas. Let α be the area of cross-section of the tube CD. Then the pressure difference between the ends of the capillary tube is caused by the mercury pellet and the excess of pressure $=\frac{mg}{\alpha}$. When the apparatus is held vertical, the mercury pellet descends down the tube, forcing a certain amount of gas through the capillary tube AB. A stop clock is started just when the upper end of the pellet crosses the mark C and stopped just when the lower end of the pellet crosses the mark D. The time interval t is noted. Then η is calculated using the formula,

$$\eta = \frac{\pi a^4 mgt}{8\alpha l(V-2v)}.$$

5.14 Bernoulli's Theorem

Statement:

The total energy of an incompressible liquid flowing from one point to another, without any friction remains constant throughout the motion.

Explanation: When an incompressible liquid flows along a path without any friction the total energy of its unit mass remains constant. The theorem follows essentially from the law of conservation of energy and is a fundamental theorem of fluid dynamics.

A liquid in flow possesses three forms of energies.

(i) **Potential energy:** Potential energy per unit mass = gh.

(ii) **Kinetic energy:** Kinetic energy per unit mass $=\frac{1}{2}v^2$.

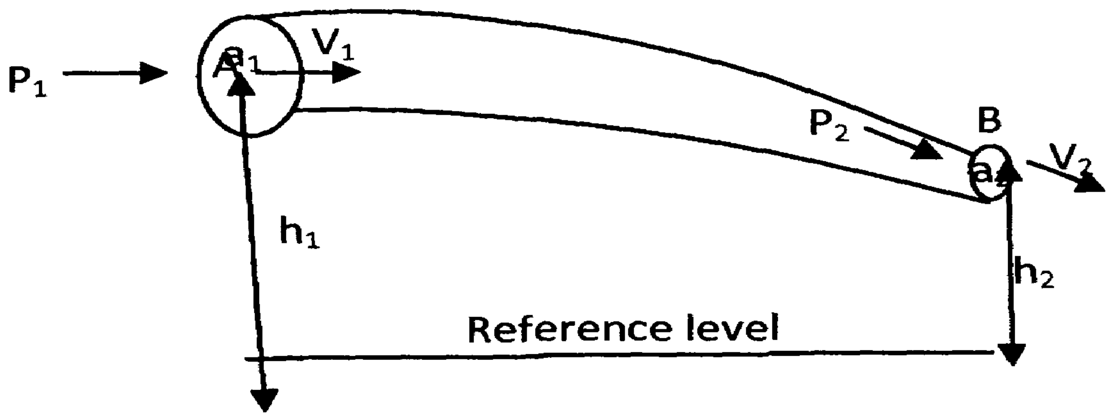
(iii) **Pressure energy:** Pressure energy per unit mass $=\frac{P}{\rho}$.

Total energy = Potential energy + Kinetic energy + Pressure energy

According to Bernoulli's Theorem, the total energy per unit mass is constant.

That is $gh + \frac{1}{2}v^2 + \frac{P}{\rho} = \text{constant}$.

Let us consider a liquid in streamed line motion through a nonuniform tube. Let a_1 and a_2 be the area of cross sections at the left end (A) and at the right end(B) (figure). Let $v_1$and $v_2$be the velocities of the liquid while entering and leaving the tube respectively.



Let P_1 be the pressure at the left end and P_2 that at the right end of the tube of flow. Since $a_1>a_2$, $v_2>v_1$.

Work done per second on the liquid entering at A is

W_1 = Force at A × Distance moved by the liquid in one second

$= P_1 \times a_1 \times v_1$ (Since, Force = Pressure × area)

$W_1 = P_1 a_1 v_1$ ..... (1)

Work done per second by the liquid leaving the tube at B is

$W_2 = P_2 a_2 v_2$ ..... (2)

The net workdone on the liquid = $W_1 - W_2$

$$W = P_1 a_1 v_1 - P_2 a_2 v_2 \quad ..... (3)$$

But, $a_1 v_1 = a_2 v_2$ (From equation of continuity)

$$\therefore W = (P_1 - P_2) a_1 v_1 \quad ..... (4)$$

The work done on liquid is used in changing its kinetic and potential energies.

Decrease in P. E = $(a_1 v_1 \rho) g (h_1 - h_2)$

Increase in K. E= $\frac{1}{2}(a_1 v_1 \rho)(v_2^2 - v_1^2)$

Hence, the total gain in the energy of the system when the liquid flows from A to B is

$$= \frac{1}{2}(a_1 v_1 \rho)(v_2^2 - v_1^2) - (a_1 v_1 \rho) g (h_1 - h_2)$$

$$\therefore (P_1 - P_2) a_1 v_1 \quad = \frac{1}{2}(a_1 v_1 \rho)(v_2^2 - v_1^2) - (a_1 v_1 \rho) g (h_1 - h_2)$$

Or $\quad P_1 + \frac{1}{2}\rho v_1^2 + \rho g h_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g h_2$

Or $\quad P + \frac{1}{2}\rho v^2 + \rho g h = \text{constant}$

In general $\quad \frac{P}{\square} + \frac{1}{2}\rho v^2 + gh = \text{constant}$ ..... (5)

This is known as Bernoulli's Theorem.

Dividing equation (5) by g,we get $\quad \frac{P}{\rho g} + \frac{v2}{2g} + h = \text{constant}$.

h is called the gravitation head

$(\frac{v2}{2g})$ is known as the velocity head

$(\frac{P}{\rho g})$ is referred to as the pressure head

Therefore at all points o a streamline of an ideal liquid the sum of the gravitation head, the velocity head and the pressure head is a constant. This is another statement of Bernoulli's Theorem.

5.15 Applications of Bernoulli's theorem

1. The Pitot's tube

This is a device used for measuring the velocity of fluids flowing through pipes. It consists of a sort of manometer tube having small aperture at the ends A and B (figure). The plane of the aperture at A is parallel to the direction of flow of the fluid while that at B is perpendicular to the direction of flow.

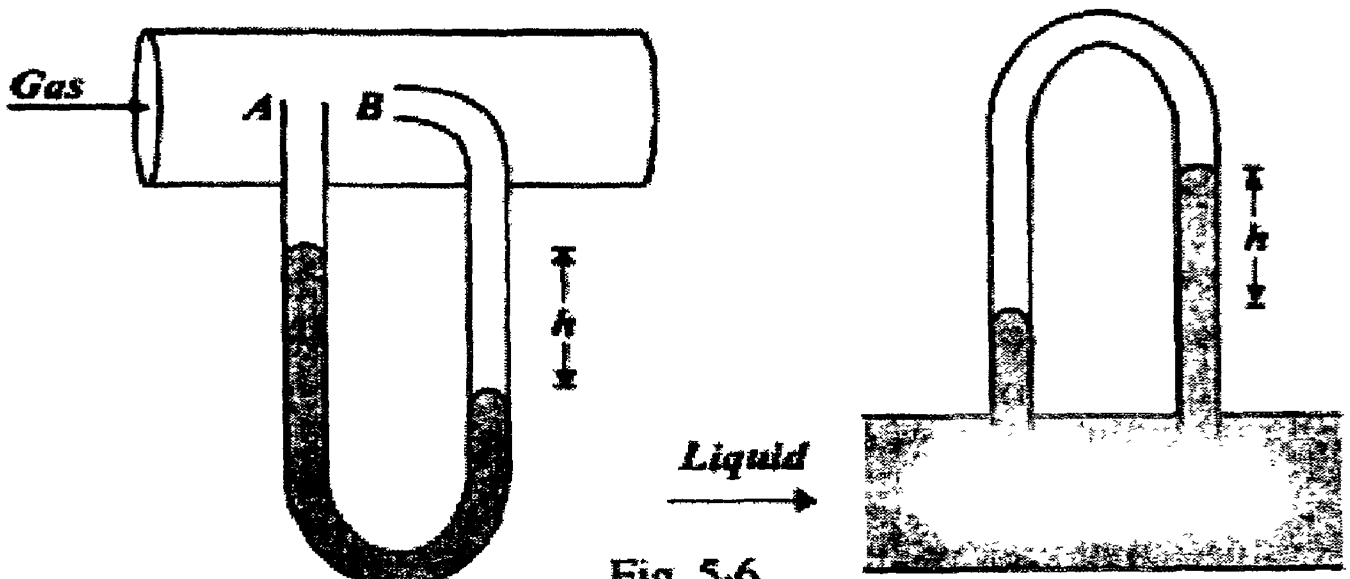


Fig. 5-6

The velocity and therefore the pressure of the fluid at A remains the same. Let this static pressure be P_1. The velocity of the fluid at B is suddenly reduced to almost zero. The pressure has a maximum value P_2. Applying Bernoulli's theorem for the horizontal flow of the fluid at the points A and B.

$$\frac{P_1}{\rho} + \frac{1}{2}v^2 = \frac{P_2}{\rho} + 0 \quad \text{............ (1)}$$

Suppose h is the manometer reading

$$P_1 + hdg = P_2 \quad \text{......... (2)}$$

Where d is the density of the manometer liquid. Form (1) and (2) we have

$$\frac{P_1}{\rho} + \frac{1}{2}v^2 = \frac{P_1 + hdg}{\rho}$$

$$\frac{v^2}{2} = \frac{hdg}{\rho}$$

$$\therefore v = \sqrt{\frac{2hgd}{\rho}}$$

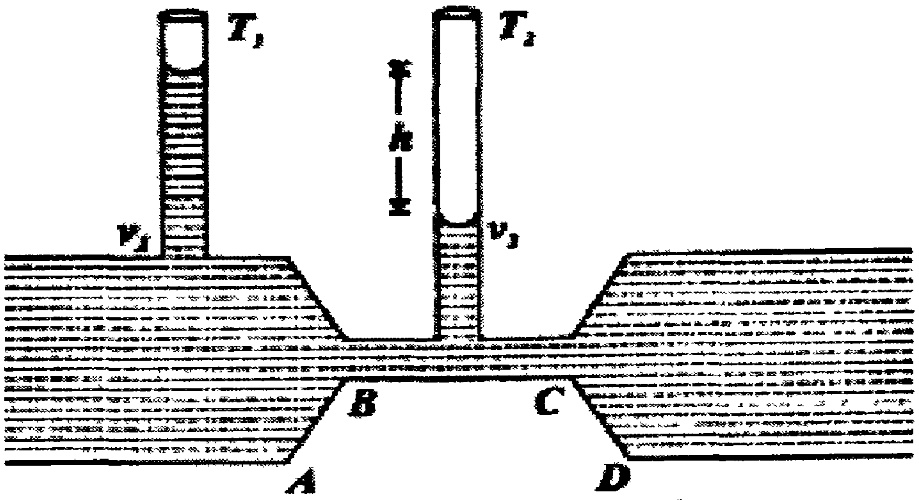
In the case of liquids d = ρ

$$\therefore v = \sqrt{2gh}$$

The device can be calibrated to read the velocity v directly.

2. Venturimeter

Venturimeter is an instrument used for measuring the rate of flow of liquids in pipes. The working of the instrument is based on Bernoulli's theorem.



It consists of three parts (i) a convergent cone AB(area of cross-section A) (ii) a horizontal tube BC of uniform diameter called the throat (area of cross-section a) and (iii) a divergent cone CD called a diffuser (figure). The instrument is connected so that it forms a part of the main pipe through which the liquid flows. Two vertical tubes T_1 and T_2 are provided to measure the pressure P_1 and P_2 in the pipe and the throat respectively. Let v_1 and v_2 be the velocities of the liquids in the pipe and throat respectively.

According to Bernoulli's theorem $\frac{v_1^2}{2} + \frac{P_1}{\rho} = \frac{v_2^2}{2} + \frac{P_2}{\rho}$

Here ρ is the density of the liquid. Since the pipe is horizontal, the gravitational head 'gh' is the same on both the tubes and is neglected.

$$\frac{P_1 - P_2}{\rho} = \frac{v_2^2 - v_1^2}{2}$$

$P_1 - P_2 = h\rho g$, where h is the difference of liquid levels in the two tubes T_1 and T_2.

$$\frac{h\rho g}{\rho} = \frac{v_2^2 - v_1^2}{2}$$

According to the equation of continuity $Av_1 = av_2 = V$

$$v_1 = \frac{V}{A} \text{ and } v_2 = \frac{V}{a}$$

Therefore $$hg = \frac{1}{2}\left[\frac{V^2}{a^2} - \frac{V^2}{A^2}\right]$$

$$hg = \frac{V^2}{2}\left[\frac{1}{a^2} - \frac{1}{A^2}\right] = \frac{V^2}{2}\left[\frac{A^2 - a^2}{a^2 A^2}\right]$$

Or $$V^2 = 2hg\left[\frac{A^2 a^2}{A^2 - a^2}\right]$$

Or $$V = Aa\sqrt{\frac{2hg}{A^2 - a^2}}$$

The height h to which the liquid rises is measured. Knowing A and a the area of cross-section of the pipe and the throat respectively the velocity of the liquid can be calculated.

5.16 Surface tension

Molecular Forces

Since surface tension is essentially a molecular phenomenon, it is better if we first have a clear idea as to what forces operate between molecules.

There are two types of molecular forces: (i) **forces of adhesion**, or adhesive forces, and (ii) **forces of cohesion** or cohesive forces.

(i) Adhesion is the force of attraction between molecules of different substances, and is different for different pairs of substances, e.g., gum has a greater adhesive force than water or alcohol.

(ii) Cohesion, on the other hand, is the force of attraction between molecules of the same substance. This force is different from the ordinary gravitational force arid does not obey the ordinary inverse square law, the force varying inversely probably as the eighth power of the distance between two molecules and thus decreases rapidly with distance, -in fact it is appreciable when the distance between two molecules is inappreciable and becomes inappreciable when the distance is appreciable. It is the greatest, in the case of solids, less in the case of liquids and the least in the case of gases, almost negligible at ordinary temperature and pressure, when the molecules lie very much further apart for it to, be appreciable. This explains at once why a solid has a definite shape, a liquid has a definite free surface and a gas has neither.

Molecular range-sphere of Influence

The maximum distance up to which the force of cohesion -between two molecules can act is called their molecular range, and is generally of the order of 10^{-7} cm in the case of solids and liquids, being different for different substances. A sphere drawn around a molecule as centre, with a radius equal to its molecular range is called the sphere of influence of the molecule. Obviously, the molecule i affected only by the molecules inside this sphere, i.e., it attracts and is, in turn, attracted by them, remaining unaffected by the molecules outside it, as they He beyond its range of attraction. Laplace (1806) and Gauss (1830) were the first to have devolved this theory of cohesive force between molecules in order to satisfactorily explain the various effects of surface tension, like capillarity etc.

Surface Tension

It is a general experience that a liquid in small quantity, free from any external force, like that due to gravity, will always assume the form of a spherical drop e.g., rain drops, small quantities of mercury placed on a clean glass plate etc.

Now, for a given volume, a sphere has the least surface area. Thus, a liquid always tends to have the least surface area. The following experiments beautifully illustrate this tendency of a liquid to decrease its surface area.

Let a drop of olive oil be placed, with the help of pipette, inside a mixture of alcohol and water, of the same density as that of the oil, but with which it does not mix. It will be found that the drop will keep on floating in the mixture with a perfectly spherical form. The reason obviously is that weight of the drop having bean exactly balanced by the up thrust of the displaced mixture, the action of gravity on it is completely eliminated and it is thus subject to the force of surface tension alone, which compels it to acquire a shape having the least surface area. And, since for a given volume, a sphere has the minimum area, the drop acquires this particular shape. This is referred to as Platen's experiment.

If we place a greased needle on a piece of blotting paper and put the paper lightly on the surface of water, the blotting paper will soon sink to the bottom, but the needle will remain floating on the surface Careful observation will show that there is a small depression formed below and around the needle, and that the free surface of water is slightly extended The weight of the needleis here supported by the tension in the depression. If one end of the needle be made to pierce the surface of water, it rapidly goes slantingly down to the bottom.

If we immerse an ordinary camel hair paint brush in water, its hair all spread out, presenting a sort of a bushy appearance, but the moment it is withdrawn, they all come closer together in a more or less compact mass, as though bound down by some sort of a contracting membrane.

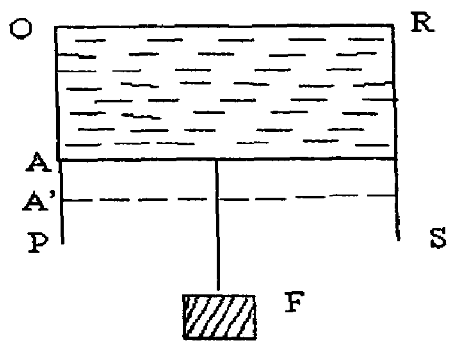
Yet another beautiful experiment, often performed for fun by junior students, is the rapid movement of a camphor scorpion on water. What they do is simply to arrange pieces of camphor together, in the shape of a scorpion, and put it on water, when, due to the reduction in the surface tension of water, on account of the camphor gradually dissolving into it, the camphor is drawn or pulled aside by the surrounding uncontaminated water of a higher surface tension. And, since we have camphor dissolving more rapidly at some points than at others, this force due to surface tension is not uniform all round, with the result that the 'scorpion' scampers about haphazardly in different directions.

If, however, the water be already contaminated with some grease etc., its surface tension may be reduced to an extent that the camphor has no further possibility of reducing it. In such a case, therefore, the movement of the camphor may altogether stop.

The above experiments clearly show that the surface of a liquid behaves as though it were covered with an elastic skin or membrane, having a natural tendency to contract, with the important difference, however, that whereas in the case of the membrane or skin, the tension increases as the skin is stretched, or its surface area is increased, in accordance with Hooke's Law, it is quite independent of the area of the surface in, the case of a liquid, unless the liquid film is reduced in thickness to less than 10^{-7} cm, when the tension in it decreases rapidly. This tension or pull in the surface of a liquid is called its surface tension, and may be defined as the force per unit length of a line drawn in the liquid surface, acting perpendicularly to it at every point t and tending to pull the surface apart along the line.

5.17 Free energy of a surface and surface tension

Take a rectangular framework of wire PQRS, (Figure), with a horizontal wire AB placed across it, free to move up and down, and form a soap- film across AQRB, by dipping it in a soap solution. The wire AB is pulled upwards by the surface tension of the film, acting perpendicularly to the wire and in the plane of the film. To keep the wire in position, therefore, a force has to be applied downwards, equal and opposite to the upwards force due to surface tension.



Let this downward force be equal to F including the weight of the wire AB, which is also acting downwards. Then, if T be the surface tension of the film, i.e., if T be

the force per unit length of the film and l, the length of the wire AB, we have upward force acting on the wire AB = 2 I.T, because the film has two surfaces and each has a surface tension T.
Since the film is in equilibrium, it is clear that 2 I.T = F.

Now, if the wire AB be pulled downwards through a small distance x into the position A'B', i.e., if the film be extended by an area l.x on each side, we have

Workdone = F ×x =$2lTx$.

The film gets cooled on being stretched, because the drawing out of the molecules from the interior against the attractive force results in a retardation of their thermal agitation, with a consequent lowering of temperature*. It, therefore, takes up heat from the atmosphere to come to its original temperature. This heat absorbed, together with the mechanical work done, forms the energy of the new surface area 2lx of the film formed.

If, therefore, E be the surface energy of the film and Q ergs of heat be absorbed per unit area of the new surface formed , we have E× $2l$x = $2l$ T.x+Q$2l$.x.

Or, E = T + Q. [Dividing throughout by 2lx]

Or, T = (E-Q) = (surface energy heat energy per unit area)

T = potential energy per unit area.

T = work done in Beating unit area of the film.

Thus, the surface tension of a liquid may be defined as the amount of work done in increasing the surface area of the liquid-film by unity, or as the mechanical part of the surface energy of the liquid film. This mechanical part of the surface energy of a liquid-film is free energy, so that, the surface tension of a liquid is equal to the free energy of the liquid film or surface.

Units and dimensions of surface tension

As we have seen above, surface tension may be expressed as force per unit length or as energy per unit area. Its units, therefore, are

(i) Dyne/cm or erg/sq. cm in the C.G.S. system.
(ii) poundal/ft or ft. poundal/sq.ft.in the F.P.S. system and
(iii) newton/metre (N/m) or joule/sq.metgre in the M.K.S.(S.I) system . And, its dimensions are those of force/length or work or energy/area.

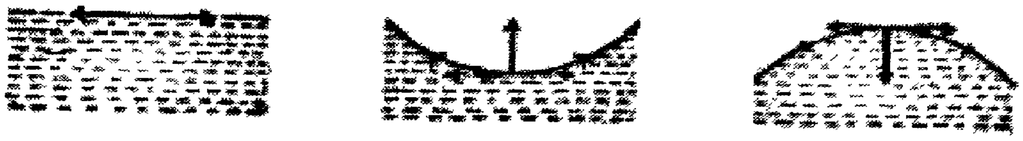
i.e., $\frac{MLT-2}{L}$ or $\frac{ML2T-2}{L2}$

both of which come to MT^{-2}

Thus, the dimension of surface tension is $ML^0\ T^{-2}$ or MT^{-2}.

Pressure Difference across a Liquid Surface

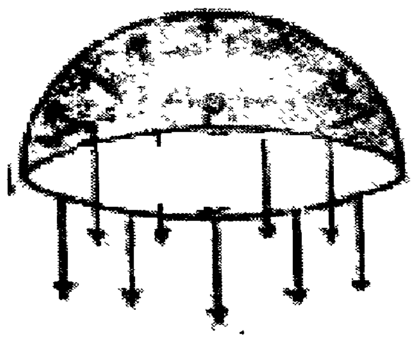
(i) Suppose the free surface of a liquid is plane, as shown in Figure (i). Then, the resultant force due to surface tension on a molecule on its surface is zero, and the cohesion- pressure is, therefore, just nominal or negligible.



(ii) If the free surface of the liquid be concave, [Figure (ii)], the resultant force on a molecule on the surface would be upwards, and the cohesion pressure is, therefore, decreased.
(iii) And, finally, if the liquid surface be convex, as shown in Figure (iii), the resultant force due to surface tension on a molecule on the surface will be directed downwards, so that the cohesion pressure is, in this case, increased.

5.18 Excess pressure inside a liquid drop

It must be clear from the above that the molecules near the surface of a drop, (which is a convex surface), experience a resultant pull inwards. The pressure inside it must, therefore, be greater than the pressure outside it. Let this excess pressure inside over the pressure outside the drop be p. Then, if r be the radius of the drop, and T, its surface tension, we have, considering the equilibrium of one-half of the drop-say, the upper half, or the upper hemisphere, the upward thrust on the plane face ABCD, (Figure), due to the excess pressure p is equal to p πr^2.



And, force due to surface tension, acting downwards on it and round its edge, is equal to T. 2π r. Since the hemisphere is in equilibrium, we have p. □r^2 = T. 2π r,

Hence, $$p = \frac{T.2\pi r}{\pi r2} = \frac{2T}{r}.$$

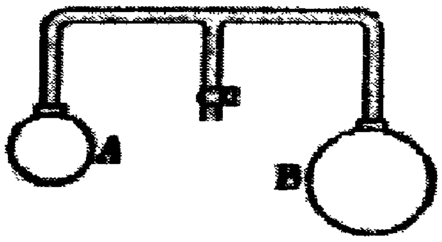
5.19 Excess pressure inside a soap bubble

If, instead of a drop of liquid, we consider a bubble, there are two surfaces to be considered, and not one, because it is like a spherical shell or a hollow sphere, having an inner and an outer surface ; so that, the force due to surface tension, in this case, is $2 \times 2 \pi r T$, (i.e., $2\pi r.T$ due to each surface).Therefore, for equilibrium of the hemisphere, we have,

$$p. \pi r^2 = 2 \times 2 \pi r T = 4 \pi r T,$$

Hence $$p = \frac{4 \pi r t}{\pi r 2} = \frac{4T}{r}$$

In case the bubble happens to lie inside a liquid as for example, an air bubble inside water, it will have only one free surface and will thus behave take a drop. Thus excess pressure inside it therefore will be the same as inside a drop, i.e., 2T/r.



Alternatively, we could obtain the above expression for excess pressure inside a drop and a bubble as follows:

Suppose, on account of the excess pressure P inside a liquid drop of radius r, the liquid surface is pushed outwards through a very small distance dr, such that there is little or no increase in volume and the excess pressure continues to remain p. Then clearly, increase in surface area of the drop $= 4 \pi (r + dr)^2 - 4\pi r^2 = 8 \pi r\, dr$

Neglecting $(dr)^2$ as a very small quantity.

∴ Increase in surface energy of the liquid surface $= T \times 8\pi r dr$.

This must obviously be equal to the work done in pushing the liquid surface outwards through dr , i.e., equal to $4\pi r^2 \times p \times dr$.

Equating the two, therefore we have $4\pi r^2\, p\, dr = 8 \pi Tr\, dr$.

Hence, $$p = \frac{2T}{r}$$

In the case of bubble, there being two surface in contact with air, the total increase in surface area of the liquid film is $2 \times 8 \pi r\, dr$ and , therefore , increase in surface energy of the film $= T \times 16 \pi r\, dr$

So that, $$4\pi r^2.p.dr = T \times 16 \pi r\, dr.$$

$$\text{Hence } p = \frac{4T}{r}$$

It will thus be seen that the excess pressure inside a drop or a bubble is inversely proportional to its radius (i.e., p * 1/r) ; so that, the smaller the bubble, the greater the excess pressure inside it.

5.20 Work done in blowing a bubble

If, for the sake of simplicity, we neglect the cooling produced when a film is stretched, the work done in blowing a bubble is easily calculated out as follows:

We know that, work done in creating a film = surface tension × area of the film formed.

If, therefore, the radius of the bubble blown be r, the area of the film forming the bubble = $2 \times 4\pi r^2$ for it has two surfaces, an inner and an outer one, each of surface area $4\pi r^2$.

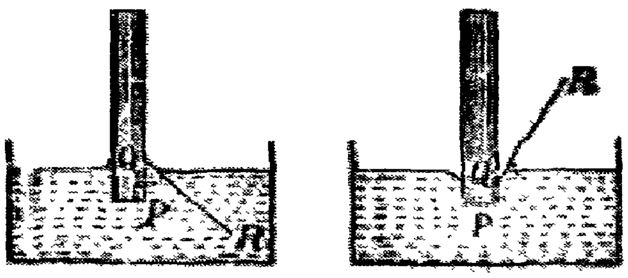
Therefore, work done in blowing the bubble = $T \times 8\pi r^2 = 8\pi r^2 T$.

5.21 Angle of contact

(i) Case of liquid in contact with a solid

When a liquid meets a solid, its surface near its plane of contact with the solid is, In general, curved. The angle between the tangent to the liquid surface at the point of contact and the solid surface, inside the liquid is called the angle of contact for that pair of solid and liquid.

This angle may have any value between 0^0 and 180^0. For most liquids and glass, it is less than 90^0; for mercury and glass it is about 140^0.It really depends upon the nature of the liquid and the solid, and is not altered by a change in the inclination of the solid.

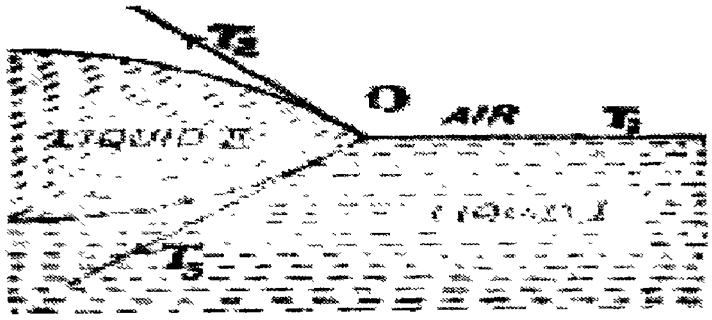


In the figures shown, [Figure (i) and (ii)], < PQR is the angle of contact. It is acute in (i) and obtuse in (ii); for, in the former case, the liquid rises up a little alongside the glass plate, dipped in the liquid, and the angle between QR, the tangent to the liquid surface and the part QP of the plate, inside the liquid, is acute, whereas, in the latter case, the liquid is depressed a little where it comes into contact with the glass plate, and the angle between the tangent PR to the liquid surface on the part QP of the plate, inside the liquid, is obtuse.

For pure water and clean glass, the angle of contact is 0^0. For ordinary water and glass it is about 18^0; and if the surface of the glass be contaminated with grease, its value may be as much as 35^0.

(ii) Case of two liquids in contact with each other and with air.

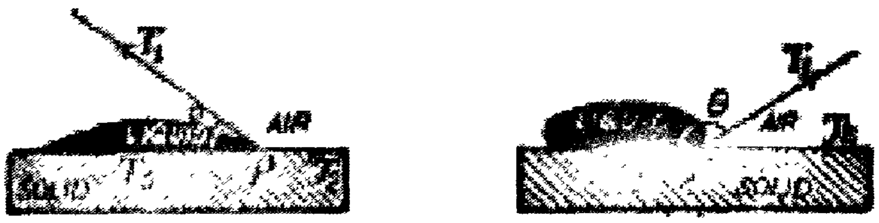
If two liquids, not miscible with each other, be brought into contact as at 0, (Figure), both being in contact with air, three surface tensions are to be taken into consideration, (a) that of the surface between air and liquid I, viz.,T_1 ; (b) that of the surface between air and liquid II, viz., T_2 ; and (c) that between aliquidI and liquid II, viz., T_3 .



For equilibrium, $T_1, T_2,$ and T_3should be represented by the three sides of a triangle, taken in order. This triangle of forces is known as Neumann's triangle. In actual practice, we come across no two pure liquids for which the Neumann's triangle may be constructed, one of the surface tensions being always greater than the other two; so that, the equilibrium condition shown in the figure is never attained. Thus, for example, in the cage of water, mercury and air, the water drop, when placed over mercury, spreads all over its surface, provided both water and mercury are pure. This is so, because the surface tension of mercury is about 550 dynes/cm. and that of water, only 75 dynes/cm. But, if the mercury surface be contaminated with grease, its surface tension decreases and some water drops may stay on it, so that, in that ease, the construction of Neumann's triangle can be possible.

(iii) Case of a solid, liquid and air in contact.

This is more important case than the previous one, for we have to consider three surface tensions, viz., T_1 for air-liquid, T_2 for air-solid and $T_3$for liquid-solid surfaces respectively, (Figure).



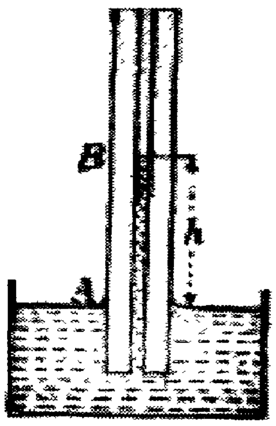
Let θ be the angle of contact of the liquid with the solid, acute, in case (a) and obtuse in case (b). For equilibrium, therefore, T_3 plus the component $T_1 \cos\theta$ of T_1 in the direction of T_3 , must be balanced by T_2, i.e. $T_3 + T_1 \cos\theta = T_2$.

Or, $T_1 \cos\theta = T_2 - T_3$, whence, $\cos\theta = (T_2 - T_3) / T_1$

Clearly, therefore, if T_2 is greater than T_3 , $\cos\Theta$ will be positive, i e., Θ will be less than 90^0 and ifT_2 is less than $T_3\cos\theta$ will be negative, and θ will lie between 90^0 and 180^0. If, however $T_2 > (T_1 + T_3)$there will be no equilibrium, and the liquid will spread over the solid, as happens when a water drop is placed over a perfectly clean plate of glass, or a grease- free mercury surface.

5.22 Rise of liquid in a capillary tube

One of the most striking effects of surface tension is to raise a liquid in a capillary tube dipped into it, a capillary tube being just a tube of a very fine bore (from the Latin word, capillus a hair). It is for this reason that surface tension is also sometimes called capillarity.



When a capillary tube is dipped in a liquid like water, which wets it and for which the angle of contact may be taken to be zero, the liquid immediately rises up into it, and if the tube be &fine one, tin shape of the liquid meniscus is spherical and concave upwards, as shown at B, (Figure).

Let r be the radius of the tube at B, the point up to which the liquid rises into it. Then, it will be practically the same as the radius of the concave meniscus, so that the excess pressure above the meniscus over that immediately below it is 2T/r,i.e., the pressure in the liquid, just below the meniscus, is less than the atmospheric pressure above it by 2T/r. And, since the pressure on the liquid surface, outside the tube, is atmospheric, the liquid will be forced up into the tube, until the hydrostatic pressure of the liquid column in the tube equals this excess pressure 2T/r. If the liquid rises to a height h, the hydrostatic pressure due to the liquid column in the tube on the surface of the liquid will clearly be h.ρ.g, where ρ is the density of the liquid. **

$\therefore 2T/r = h.\ \rho.\ g$ Or, $2T = r.h.\ \rho.\ g.$

Hence, $T = \frac{r.h.\rho.g}{2}$

Thus, knowing r, h, ρ and g, the surface tension T of the liquid can be easily determined.

EXERCISE V

PART A

1. Define coefficient of viscosity.
2. Define critical velocity.
3. Write a note on streamline and turbulent flow.
4. What is Reynolds's number?
5. Write down the Poiseuille's formula for the rate of flow of a liquid.
6. Define surface tension.
7. Write a note on angle of contact.
8. What is meant by capillary rise? Give the reason for that.
9. State Bernoulli's theorem.
10. Write down the applications of Bernoulli's theorem.
11. What is meant by terminal velocity and write an expression for it.

PART B

1. Derive Poiseuille's formula for the rate of flow of a liquid through capillary tube.
2. Derive Stoke's formula for the viscous force by the method of dimensions.
3. Explain Ostwald's viscometer to determine coefficient of viscosity of a gas.
4. Derive an expression for the excess pressure inside a soap bubble.
5. Derive an expression for work done in blowing a soap bubble.
6. Explain the working of Pitot's tube with theory.
7. Describe function of Venturimeter with theory.
8. Determine the surface tension of a liquid by capillary rise method.

PART C

1. Determine the coefficient of viscosity of a liquid by capillary flow method with theory.
2. Explain Rankin's method to find the viscosity of a gas.
3. Determine the coefficient of viscosity of a highly viscous liquid by Stoke's method.
4. State and prove Bernoulli's theorem.

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